

Exercise 6.1

Q.1 Find the H.C.F of the following expressions.

(i) $39x^7y^3z$ and $91x^5y^6z^7$

Solution:

$$39x^7y^3z = 3 \times 13 \times x.x.x.x.x.x.x.y.y.y.z$$

$$91x^5y^6z^7 = 7 \times 13 \times x.x.x.x.x.y.y.y.y.y.z.z.z.z.z.z.z$$

$$\text{H.C.F} = 13 \times x.x.x.x.x.y.y.y.z$$

$$\text{H.C.F} = 13x^5y^2z$$

(ii) $102xy^2z, 85x^2yz$ and $187xyz^2$

Solution:

$$102xy^2z = 2 \times 3 \times 17 \times x.y.y.z$$

$$85x^2yz = 5 \times 17 \times x.x.y.z$$

$$187xyz^2 = 11 \times 17 \times x.y.z.z$$

$$\text{H.C.F} = 17xyz$$

Q.2 Find the H.C.F of the following expression by factorization.

(i) $x^2 + 5x + 6, x^2 - 4x - 12$

Solution: $x^2 + 5x + 6, x^2 - 4x - 12$

Factorizing $x^2 + 5x + 6$

$$= x^2 + 3x + 2x + 6$$

$$= x(x+3) + 2(x+3)$$

$$= (x+3)(x+2)$$

Factorizing $x^2 - 4x - 12$

$$= x^2 - 6x + 2x - 12$$

$$= x(x-6) + 2(x-6)$$

$$= (x-6)(x+2)$$

So,

$$\text{H.C.F} = (x+2)$$

(ii) $x^2 - 27, x^2 + 6x - 27, 2x^2 - 18$

Solution: $x^2 - 27, x^2 + 6x - 27, 2x^2 - 18$

Factorizing $x^3 - 27$

$$= (x)^3 - (3)^3$$

$$= (x-3) \left[(x)^2 + (x)(3) + (3)^2 \right]$$

$$= (x-3)(x^2 + 3x + 9)$$

Factorizing $x^2 + 6x - 27$

$$= x^2 + 9x - 3x - 27$$

$$= x(x+9) - 3(x+9)$$

$$= (x+9)(x-3)$$

Factorizing $2x^2 - 18$

$$= 2(x^2 - 9)$$

$$= 2 \left[(x)^2 - (3)^2 \right]$$

$$= 2(x-3)(x+3)$$

So,

$$\text{H.C.F} = (x-3)$$

(iii) $x^3 - 2x^2 + x, x^2 + 2x - 3, x^2 + 3x - 4$

Factorizing $x^3 - 2x^2 + x$

$$= x(x^2 - 2x + 1)$$

$$= x(x^2 - x - x + 1)$$

$$= x \left[x(x-1) - 1(x-1) \right]$$

$$= x(x-1)(x-1)$$

Factorizing $x^2 + 2x - 3$

$$= x^2 + 3x - x - 3$$

$$= x(x+3) - 1(x+3)$$

$$= (x+3)(x-1)$$

Factorizing $x^2 + 3x - 4$

$$= x^2 + 4x - x - 4$$

$$= x(x+4) - 1(x+4)$$

$$= (x+4)(x-1)$$

So,

$$\text{H.C.F} = (x-1)$$

(iv) $18(x^3 - 9x^2 + 8x), 24(x^2 + 3x + 2)$

Solution: $18(x^3 - 9x^2 + 8x), 24(x^2 + 3x + 2)$

Factorizing $18(x^3 - 9x^2 + 8x)$

$$= 6 \times 3 \times x(x^2 - 9x + 8)$$

$$\begin{aligned}
&= 6 \times 3 \times x(x^2 - 8x - x + 8) \\
&= 6 \times 3 \times x[x(x-8) - 1(x-8)] \\
&= 6 \times 3 \times x(x-8)(x-1)
\end{aligned}$$

Factorizing $24(x^2 + 3x + 2)$

$$\begin{aligned}
&= 6 \times 4(x^2 - 3x + 2) \\
&= 6 \times 4(x^2 - 2x - x + 2) \\
&= 6 \times 4[x(x-2) - 1(x-2)] \\
&= 6 \times 4(x-2)(x-1)
\end{aligned}$$

So,

$$\text{H.C.F} = 6(x-1)$$

(v) $36(3x^4 + 5x^2 - 2x^2), 54(27x^4 - x)$

Factorizing $36(3x^4 + 6x^3 - 2x^2)$

$$\begin{aligned}
&= 3 \times 3 \times 2 \times 2 \times x^2(3x^2 + 5x - 2) \\
&= 3 \times 3 \times 2 \times 2 \times x^2(3x^2 + 6x - x - 2) \\
&= 3 \times 3 \times 2 \times 2 \times x^2[3x(x+2) - 1(x+2)] \\
&= 3 \times 3 \times 2 \times 2 \times x^2(x+2)(3x-1)
\end{aligned}$$

Factorizing $54(27x^4 - x)$

$$\begin{aligned}
&= 3 \times 3 \times 3 \times 2 \times x(27x^3 - 1) \\
&= 3 \times 3 \times 3 \times 2 \times x[(3x)^3 - (1)] \\
&= 3 \times 3 \times 3 \times 2 \times x(3x-1)[(3x)^2 + (3x)(1) + (1)^2] \\
&= 3 \times 3 \times 3 \times 2 \times x(3x-1)(9x^2 + 3x + 1)
\end{aligned}$$

So,

$$\text{H.C.F} = 3 \times 3 \times 2 \times x(3x-1)$$

$$= 18x(3x-1)$$

Q.3 Find the H.C.F of the following by division method.

(i) $x^3 + 3x^2 - 16x + 12, x^3 + x^2 - 10x + 8$

Solution: $x^3 + 3x^2 - 16x + 12, x^3 + x^2 - 10x + 8$

$$\begin{array}{r} 1 \\ x^3 + x^2 - 10x + 8 \overline{) x^3 + 3x^2 - 16x + 12} \\ \underline{\pm x^3 \pm x^2 \mp 10x \pm 8} \\ 2x^2 - 6x + 4 \\ 2(x^2 - 3x + 2) \end{array}$$

$$\begin{array}{r} x + 4 \\ x^2 - 3x + 2 \overline{) \cancel{x^3} + x^2 - 10x + 8} \\ \underline{\cancel{-x^3} \mp 3x^2 \pm 2x} \\ 4x^2 - 12x + 8 \\ \underline{\pm 4x^2 \mp 12x \pm 8} \\ \times \end{array}$$

H.C.F = $(x^2 - 3x + 2)$

(ii) $x^4 + x^3 - 2x^2 + x - 3, 5x^3 + 3x^2 - 17x + 6$

Solution: $x^4 + x^3 - 2x^2 + x - 3, 5x^3 + 3x^2 - 17x + 6$

$$\begin{array}{r} x + 2 \\ 5x^3 + 3x^2 - 17x + 6 \overline{) x^4 + x^3 - 2x^2 + x - 3} \\ \times 5 \\ \hline 5x^4 + 5x^3 - 10x^2 + 5x - 15 \\ \underline{\pm 5x^4 \pm 3x^3 \mp 17x^2 \pm 6x} \\ 2x^3 + 7x^2 - x - 15 \\ \times 5 \\ \hline 10x^3 + 35x^2 - 5x - 75 \\ \underline{\pm 10x^3 \pm 6x^2 \mp 34x \pm 12} \\ 29x^2 + 29x - 87 \\ 29(x^2 + x - 3) \\ \hline 5x - 2 \\ x^2 + x - 3 \overline{) 5x^3 + 3x^2 - 17x + 6} \\ \underline{\pm 5x^3 \pm 5x^2 \mp 15x} \\ -2x^2 - 2x + 6 \\ \underline{\mp 2x^2 \mp 2x \pm 6} \\ \times \end{array}$$

H.C.F = $(x^2 + x - 3)$

$$(iii) \quad 2x^5 - 4x^4 - 6x, x^5 + x^4 - 3x^3 - 3x^2$$

$$\begin{array}{r}
 \frac{1}{2x^5 - 4x^4 - 6x} \overline{) x^5 + x^4 - 3x^3 - 3x^2} \\
 \underline{\times 2} \\
 2x^5 + 2x^4 - 6x^3 - 6x^2 \\
 \underline{-2x^5 \mp 4x^4 \qquad \mp 6x} \\
 6x^4 - 6x^3 - 6x^2 + 6x \\
 6(x^4 - x^3 - x^2 + x) \\
 \\
 x^4 - x^3 - x^2 + x \overline{) 2x^5 - 4x^2 - 6x} \\
 \underline{\pm 2x^5 \pm 2x^2 \quad \mp 2x^4 \mp 2x^3} \\
 -2x^4 + 2x^3 - 2x^2 - 6x \\
 \underline{\mp 2x^4 \pm 2x^3 \pm 2x^2 \mp 2x} \\
 -4x^2 - 4x \\
 -4(x^2 + x) \\
 \\
 x^2 + x \overline{) x^4 - x^3 - x^2 + x} \\
 \underline{-x^4 \pm x^3} \\
 -2x^3 - x^2 + x \\
 \underline{\mp 2x^3 \mp 2x^2} \\
 x^2 + x \\
 \underline{\pm x^2 \pm x} \\
 \times
 \end{array}$$

$$\text{H.C.F} = x^2 + x$$

Q.4 Find the L.C.M of the following expressions.

(i) $39x^7y^3z$ and $91x^5y^6z^7$

Solution:

$$39x^7y^3z = 3 \times 13 \times x.x.x.x.x.x.x.y.y.y.z$$

$$91x^5y^6z^7 = 7 \times 13 \times x.x.x.x.x.y.y.y.y.y.y.z.z.z.z.z.z.z$$

$$\text{Common} = 13x^5y^3z$$

$$\text{Uncommon} = 3 \times 7 \times x^2y^3z^6$$

$$= 21x^2y^3z^6$$

$$\text{L.C.M} = \text{common factors} \times \text{uncommon factors}$$

$$= 13x^5y^3z \times 21x^2y^3z^6$$

$$273x^7y^6z^7$$

(ii) $102xy^2z, 85x^2yz$ and $187xyz^2$

Solution:

$$102xy^2z = 2 \times 3 \times 17 \cdot x \cdot y \cdot y \cdot z$$

$$85x^2yz = 5 \times 17 \times x \cdot x \cdot y \cdot z$$

$$187xyz^2 = 11 \times 17 \cdot x \cdot y \cdot z \cdot z$$

$$\text{Common} = 17xyz$$

$$\text{Uncommon} = 2 \times 3 \times 5 \times 11 \cdot xyz$$

$$= 330xyz$$

$$\text{L.C.M} = \text{common} \times \text{uncommon}$$

$$= 17xyz \times 330xyz$$

$$= 5610x^2y^2z^2$$

Q.5 Find the L.C.M of the following by factorizing.

(i) $x^2 - 25x + 100$ and $x^2 - x - 20$

Solution: $x^2 - 25x + 100$ and $x^2 - x - 20$

Factorizing $x^2 - 25x + 100$

$$= x^2 - 20x - 5x + 100$$

$$= x(x - 20) - 5(x - 20)$$

$$= (x - 20)(x - 5)$$

Factorizing $x^2 - x - 20$

$$= x^2 - 5x + 4x - 20$$

$$= x(x - 5) + 4(x - 5)$$

$$= (x - 5)(x + 4)$$

So,

$$\text{L.C.M} = (x - 5)(x + 4)(x - 20)$$

(ii) $x^2 + 4x + 4, x^2 - 4, 2x^2 + x - 6$

Solution: $x^2 + 4x + 4, x^2 - 4, 2x^2 + x - 6$

Factorizing $x^2 + 4x + 4$

$$= x^2 + 2x + 2x + 4$$

$$= x(x + 2) + 2(x + 2)$$

$$= (x + 2)(x + 2)$$

Factorizing $x^2 - 4$

$$= (x)^2 - (2)^2$$

$$= (x - 2)(x + 2)$$

Factorizing $2x^2 + x - 6$

$$= 2x^2 + 4x - 3 - 6$$

$$= 2x(x + 2) - 3(x + 2)$$

$$= (x + 2)(2x - 3)$$

So,

$$\begin{aligned} \text{L.C.M} &= (x+2)(x+2)(x-2)(2x-3) \\ &= (x+2)^2(x-2)(2x-3) \end{aligned}$$

(iii) $2(x^4 - y^4), 3(x^3 + 2x^2y - xy^2 - 2y^3)$

Factorizing $2(x^4 - y^4)$

$$= 2 \left[(x^2)^2 - (y^2)^2 \right]$$

$$= 2(x^2 + y^2)(x^2 - y^2)$$

$$= 2(x^2 + y^2)(x+y)(x-y)$$

Factorizing $3(x^3 + 2x^2y - xy^2 - 2y^3)$

$$= 3 \left[x^2(x+2y) - y^2(x+2y) \right]$$

$$= 3(x+2y)(x^2 - y^2)$$

$$= 3(x+2y)(x+y)(x-y)$$

So,

$$\text{L.C.M} = (x+y)(x-y)(x^2 + y^2)(x+2y) \times 2 \times 3$$

$$= 6(x+y)(x-y)(x^2 + y^2)(x+2y)$$

$$= 6(x+2y)(x^4 - y^4)$$

(iv) $4(x^4 - 1), 6(x^3 - x^2 - x + 1)$

Solution: $4(x^4 - 1), 6(x^3 - x^2 - x + 1)$

Factorizing $4(x^4 - 1)$

$$= 2 \times 2 \left[(x^2)^2 - (1)^2 \right]$$

$$= 2 \times 2(x^2 + 1)(x^2 - 1)$$

$$= 2 \times 2(x^2 + 1)(x+1)(x-1)$$

$$= 6(x^3 - x^2 - x + 1)$$

$$= 2 \times 3 \left[x^2(x-1) - 1(x-1) \right]$$

$$= 2 \times 3 \left[(x-1)(x^2 - 1) \right]$$

$$= 2 \times 3(x-1)(x-1)(x+1)$$

$$\text{L.C.M} = 2 \times 2 \times 3(x-1)(x+1)(x-1)(x^2 + 1)$$

$$= 12(x-1)^2(x+1)(x^2 + 1)$$

$$= 12(x-1)(x^4 - 1)$$

Q.6 For what value of k is $(x+4)$ the H.C.F of $x^2 + x - (2k+2)$ and $2x^2 + kx - 12$?

Solution:

$$P(x) = x^2 + x - (2k+2)$$

Since $x+4$ is H.C.F of $P(x)$ and $q(x)$

$\therefore x+4$ is a factor of $P(x)$

$$\text{Hence } P(-4) = 0$$

$$x^2 + x - (2k+2) = 0$$

By putting the value of x

$$(-4)^2 + (-4) - (2k+2) = 0$$

$$16 - 4 - 2k - 2 = 0$$

$$-2k + 10 = 0$$

$$2k = 10$$

$$k = \frac{10}{2}$$

$$k = 5$$

Q.7 If $(x+3)(x-2)$ is the H.C.F of $P(x) = (x+3)(2x^2 - 3x + k)$ and $q(x) = (x-2)(3x^2 + 7x - 1)$ the find k and l

Solution: $(x-2)(x+3)$ will divide $P(x) = (x+3)(2x^2 - 3x + K)$

$(x-2)(x+3)$ will divide $P(x) = (x+3)(2x^2 - 3x + K)$

$$x-2=0$$

$$x=2$$

$$P(2) = (2+3)(2(2)^2 - 3(2) + K)$$

$$P(2) = (5)(2 \times 4 - 6 + K)$$

$$P(2) = 5(8 - 6 + K)$$

$$P(2) = 5(2 + K)$$

Remainder is equal to zero

$$5(2 + K) = 0$$

$$2 + K = \frac{0}{5}$$

$$2 + K = 0$$

$$K = -2$$

$$q(x) = (x-2)(3x^2 + 7x - 1)$$

$(x-2)(x+3)$ will be divide $q(x) = (x-2)(3x^2 + 7x - 1)$

$$x+3=0$$

$$x=-3$$

$$q(-3) = (-3-2)[3(-3)^2 + 7(-3) - 1]$$

$$q(-3) = (-5)[3(9) - 21 - 1]$$

$$q(-3) = (-5)[27 - 21 - l]$$

$$q(-3) = (-5)(6 - l)$$

Remainder is equal to zero

$$-5(6 - l) = 0$$

$$6 - l = 0$$

$$l = 6$$

Q.8 The L.C.M and H.C.F of two polynomials $P(x)$ and $q(x)$ are $2(x^4 - 1)$ and $(x+1)(x^2 + 1)$ respectively. If

$$P(x) = x^3 + x^2 + x + 1, \text{ find } q(x)$$

Solution: $\therefore P(x) \times q(x) = \text{L.C.M} \times \text{H.C.F}$

$$\therefore P(x) \times q(x) = \text{L.C.M} \times \text{H.C.F}$$

$$q(x) = \frac{\text{L.C.M} \times \text{H.C.F}}{P(x)}$$

By putting the values

$$q(x) = \frac{2(x^4 - 1)(x+1)(x^2 + 1)}{x^3 + x^2 + x + 1}$$

$$q(x) = \frac{2(x^4 - 1)(x+1)(x^2 + 1)}{x^2(x+1) + 1(x+1)}$$

$$q(x) = \frac{2(x^4 - 1)(\cancel{x+1})(\cancel{x^2+1})}{(\cancel{x+1})(\cancel{x^2+1})}$$

$$q(x) = 2(x^4 - 1)$$

Q.9 Let $p(x) = 10(x^2 - 9)(x^2 - 3x + 2)$ and $q(x) = 10x(x+3)(x-1)^2$. if the H.C.F of

$p(x), q(x)$ is $10(x+3)(x-1)$, Find their L.C.M

Solutions: $p(x) \times q(x) = \text{L.C.M} \times \text{H.C.F}$

$$p(x) \times q(x) = \text{L.C.M} \times \text{H.C.F}$$

$$\text{L.C.M} = \frac{p(x) \times q(x)}{\text{H.C.F}}$$

By putting the values

$$\text{L.C.M} = \frac{10(x^2 - 9)(x^2 - 3x + 2) \times 10x(\cancel{x+3})(x-1)^2}{10(\cancel{x+3})(\cancel{x-1})}$$

$$\text{L.C.M} = 10x(x^2 - 9)(x^2 - 3x + 2)(x-1)$$

Q.10 Let the product of L.C.M and H.C.F of two polynomial be $(x+3)^2(x-2)(x+5)$.

If one polynomial is $(x+3)(x-2)$ and the second polynomial is $x^2+kx+15$, find the value of k.

Solution: $p(x) \times q(x) = L.C.M \times H.C.F$

$$p(x) \times q(x) = L.C.M \times H.C.F$$

By putting the values

$$(x+3)(x-2)(x^2+kx+15) = (x+3)^2(x-2)(x+5)$$

$$x^2+kx+15 = \frac{(x+3)^{\cancel{2}}(\cancel{x-2})(x+5)}{(\cancel{x+3})(\cancel{x-2})}$$

$$x^2+kx+15 = (x+3)(x+5)$$

$$x^2+kx+15 = x^2+8x+15$$

$$kx = \cancel{x^2} + 8x + \cancel{15} - \cancel{x^2} - \cancel{15}$$

$$kx = 8x$$

$$k = \frac{8\cancel{x}}{\cancel{x}}$$

$$k = 8$$

Q.11 Waqas wishes to distribute 128 bananas and also 176 apples equally among a certain number of children. Find the highest number of children who can get fruit in this way.

Solution

$$\begin{array}{r} 1 \\ 128 \overline{)176} \\ \underline{128} \end{array}$$

$$\begin{array}{r} 2 \\ 48 \overline{)128} \\ \underline{-96} \end{array}$$

$$\begin{array}{r} 32 \\ 32 \overline{)48} \\ \underline{-32} \end{array}$$

$$\begin{array}{r} 2 \\ 16 \overline{)32} \\ \underline{-32} \\ 0 \end{array}$$

Highest no. of children = 16

Exercise 6.2

Q.1 Simplify each of the following as a rational expression.

(i) $\frac{x^2 - x - 6}{x^2 - 9} + \frac{x^2 + 2x - 24}{x^2 - x - 12}$

Solution: $\frac{x^2 - x - 6}{x^2 - 9} + \frac{x^2 + 2x - 24}{x^2 - x - 12}$

$$= \frac{x^2 - x - 6}{x^2 - 9} + \frac{x^2 + 2x - 24}{x^2 - x - 12}$$

$$= \frac{x^2 - 3x - 2x - 6}{(x)^2 - (3)^2} + \frac{x^2 + 6x - 4x - 24}{x^2 - x - 12}$$

$$= \frac{x(x-3) + 2(x-3)}{(x-3)(x+3)} + \frac{x(x+6) - 4(x+6)}{x(x-4) + 3(x-4)}$$

$$= \frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x+3)} + \frac{(x+6)\cancel{(x-4)}}{\cancel{(x-4)}(x+3)}$$

$$= \frac{(x+2)}{(x+3)} + \frac{(x+6)}{(x+3)}$$

$$= \frac{x+2+x+6}{x+3}$$

$$= \frac{8+2x}{x+3}$$

$$= \frac{2(x+4)}{x+3}$$

Q.2 $\left[\frac{x+1}{x-1} - \frac{x-1}{x+1} - \frac{4x}{x^2+1} \right] + \frac{4x}{x^4-1}$

Solution: $\left[\frac{x+1}{x-1} - \frac{x-1}{x+1} - \frac{4x}{x^2+1} \right] + \frac{4x}{x^4-1}$

$$= \left[\frac{x+1}{x-1} - \frac{x-1}{x+1} - \frac{4x}{x^2+1} \right] + \frac{4x}{x^4-1}$$

$$= \left[\frac{(x+1)^2 - (x-1)^2}{(x-1)(x+1)} - \frac{4x}{x^2+1} \right] + \frac{4x}{x^4-1}$$

$$= \left[\frac{x^2 + 2x + 1 - (x^2 + 1 - 2x)}{(x-1)(x+1)} - \frac{4x}{x^2+1} \right] + \frac{4x}{x^4-1}$$

$$\begin{aligned}
&= \left[\frac{\cancel{x^2} + 2x\cancel{1} - \cancel{x^2}\cancel{1} + 2x}{x^2 - 1} - \frac{4x}{x^2 + 1} \right] + \left[\frac{4x}{x^4 - 1} \right] \\
&= \left[\frac{4x}{x^2 - 1} - \frac{4x}{x^2 + 1} \right] + \frac{4x}{x^4 - 1} \\
&= \left[\frac{4x(x^2 + 1) - 4x(x - 1)}{(x^2 - 1)(x^2 + 1)} \right] + \frac{4x}{x^4 - 1} \\
&= \left[\frac{4x^3 + 4x - 4x^3 + 4x}{x^4 - 1} \right] + \frac{4x}{x^4 - 1} \\
&= \frac{8x}{x^4 - 1} + \frac{4x}{x^4 - 1} \\
&= \frac{8x + 4x}{x^4 - 1} \\
&= \frac{12x}{x^4 - 1}
\end{aligned}$$

Q.3

$$\frac{1}{x^2 - 8x + 15} + \frac{1}{x^2 - 4x + 3} - \frac{2}{x^2 - 6x + 5}$$

Solution: $\frac{1}{x^2 - 8x + 15} + \frac{1}{x^2 - 4x + 3} - \frac{2}{x^2 - 6x + 5}$

$$= \frac{1}{x^2 - 8x + 15} + \frac{1}{x^2 - 4x + 3} - \frac{2}{x^2 - 6x + 5}$$

$$= \frac{1}{x^2 - 3x - 5x + 15} + \frac{1}{x^2 - 3x - 1x + 3} - \frac{2}{x^2 - 5x - x + 5}$$

$$= \frac{1}{x(x-3) - 5(x-3)} + \frac{1}{x(x-3) - 1(x-3)} - \frac{2}{x(x-5) - 1(x-5)}$$

$$= \frac{1}{(x-3)(x-5)} + \frac{1}{(x-3)(x-1)} - \frac{2}{(x-5)(x-1)}$$

$$= \frac{(x-1) + (x-5) - 2(x-3)}{(x-3)(x-5)(x-1)}$$

$$= \frac{\cancel{x} - \cancel{1} + \cancel{x} - \cancel{5} - 2\cancel{x} + \cancel{6}}{(x-3)(x-5)(x-1)}$$

$$= \frac{0}{(x-3)(x-5)(x-1)}$$

$$= 0$$

Q.4 $\frac{(x+2)(x+3)}{x^2-9} + \frac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$

Solution: $\frac{(x+2)(x+3)}{x^2-9} + \frac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$

$$= \frac{(x+2)(x+3)}{x^2-9} + \frac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$$

$$= \frac{(x+2)(x+3)}{(x)^2-(3)^2} + \frac{(x+2)[2(x^2-16)]}{(x-4)(x^2-3x+2x-6)}$$

$$= \frac{(x+2)(\cancel{x+3})}{(x-3)(\cancel{x+3})} + \frac{(x+2)[2(x)^2-(4)^2]}{(x-4)[x(x-3)+2(x-3)]}$$

$$= \frac{(x+2)}{(x-3)} + \frac{(\cancel{x+2})[2(x+4)(\cancel{x-4})]}{(\cancel{x-4})(x-3)(\cancel{x+2})}$$

$$= \frac{(x+2)}{(x-3)} + \frac{2(x+4)}{(x-3)}$$

$$= \frac{x+2}{x-3} + \frac{2x+8}{x-3}$$

$$= \frac{x+2+2x+8}{x-3}$$

$$= \frac{3x+10}{x-3}$$

Q.5 $= \frac{x+3}{2x^2+9x+9} + \frac{1}{2(2x-3)} - \frac{4x}{4x^2-9}$

Solution: $= \frac{x+3}{2x^2+9x+9} + \frac{1}{2(2x-3)} - \frac{4x}{4x^2-9}$

$$= \frac{x+3}{2x^2+9x+9} + \frac{1}{2(2x-3)} - \frac{4x}{4x^2-9}$$

$$= \frac{x+3}{2x^2+6x+3x+9} + \frac{1}{2(2x-3)} - \frac{4x}{(2x)^2-(3)^2}$$

$$= \frac{x+3}{2x(x+3)+3(x+3)} + \frac{1}{2(2x-3)} - \frac{4x}{(2x-3)(2x+3)}$$

$$= \frac{(\cancel{x+3})}{(\cancel{x+3})(2x+3)} + \frac{1}{2(2x-3)} - \frac{4x}{(2x-3)(2x+3)}$$

$$= \frac{1}{2x+3} + \frac{1}{2(2x-3)} - \frac{4x}{(2x-3)(2x+3)}$$

$$= \frac{2(2x-3) + (2x+3) - 4x \times 2}{2(2x-3)(2x+3)}$$

$$= \frac{4x-6+2x+3-8x}{2(2x-3)(2x+3)}$$

$$\begin{aligned}
&= \frac{-2x-3}{2(2x-3)(2x+3)} \\
&= \frac{-1(\cancel{2x+3})}{2(2x-3)(\cancel{2x+3})} \\
&= \frac{-1}{2(2x-3)}
\end{aligned}$$

Q.6 $A - \frac{1}{A}$, Where $A = \frac{a+1}{a-1}$

Solution: $A - \frac{1}{A}$, Where $A = \frac{a+1}{a-1}$

$$= A - \frac{1}{A} = ?$$

Put the value of A

$$\begin{aligned}
&= \frac{a+1}{a-1} - \frac{a-1}{a+1} \\
&= \frac{(a+1)^2 - (a-1)^2}{(a-1)(a+1)} \\
&= \frac{a^2 + 2a + 1 - (a^2 - 2a + 1)}{a^2 - 1} \\
&= \frac{\cancel{a^2} + 2a + 1 - \cancel{a^2} + 2a - 1}{a^2 - 1} \\
&= \frac{4a}{a^2 - 1}
\end{aligned}$$

Q.7 $\left[\frac{x-1}{x-2} + \frac{2}{2-x} \right] - \left[\frac{x+1}{x+2} + \frac{4}{4-x^2} \right]$

Solution: $\left[\frac{x-1}{x-2} + \frac{2}{2-x} \right] - \left[\frac{x+1}{x+2} + \frac{4}{4-x^2} \right]$

$$\begin{aligned}
&= \left[\frac{x-1}{x-2} + \frac{2}{2-x} \right] - \left[\frac{x+1}{x+2} + \frac{4}{4-x^2} \right] \\
&= \left[\frac{x-1}{x-2} + \frac{2}{-x+2} \right] - \left[\frac{x+1}{x+2} + \frac{4}{-x^2+4} \right] \\
&= \left[\frac{x-1}{x-2} - \frac{2}{x-2} \right] - \left[\frac{x+1}{x+2} + \frac{4}{-(x^2-4)} \right] \\
&= \left[\frac{x-1}{x-2} - \frac{2}{x-2} \right] - \left[\frac{x+1}{x+2} - \frac{4}{x^2-4} \right]
\end{aligned}$$

$$\begin{aligned}
&= \left[\frac{x-1-2}{x-2} \right] - \left[\frac{x+1}{x+2} - \frac{4}{(x+2)(x-2)} \right] \\
&= \frac{(x-3)}{(x-2)} - \frac{(x+1)(x-2)-4}{(x+2)(x-2)} \\
&= \frac{x-3}{x-2} - \frac{x^2-x-2-4}{(x+2)(x-2)} \\
&= \frac{x-3}{x-2} - \frac{x^2-x-6}{(x-2)(x+2)} \\
&= \frac{x-3}{x-2} - \frac{x^2-3x+2x-6}{(x-2)(x+2)} \\
&= \frac{x-3}{x-2} - \frac{x(x-3)2(x-3)}{(x-2)(x+2)} \\
&= \frac{x-3}{x-2} - \frac{(x-3)(\cancel{x+2})}{(x-2)(\cancel{x+2})} \\
&= \frac{\cancel{x-3}}{\cancel{x-2}} - \frac{\cancel{x-3}}{\cancel{x-2}}
\end{aligned}$$

0 Ans

Q.8 What rational number should be subtracted from

$$= \frac{2x^2 + 2x - 7}{x^2 + x - 6} \text{ to get } \frac{x-1}{x-2}$$

Solution: let required rational number be $P(x)$

According to condition

$$\frac{2x^2 + 2x - 7}{x^2 + x - 6} - P(x) = \frac{x-1}{x-2}$$

$$P(x) = \frac{2x^2 + 2x - 7}{x^2 + x - 6} - \frac{x-1}{x-2}$$

$$= \frac{2x^2 + 2x - 7}{x^2 + 3x - 2x - 6} - \frac{x-1}{x-2}$$

$$= \frac{2x^2 + 2x - 7}{x(x+3) - 2(x+3)} - \frac{x-1}{x-2}$$

$$= \frac{2x^2 + 2x - 7}{(x+3)(x-2)} - \frac{x-1}{x-2}$$

$$= \frac{2x^2 + 2x - 7 - (x-1)(x+3)}{(x+3)(x-2)}$$

$$= \frac{2x^2 + 2x - 7 - (x^2 + 2x - 3)}{(x+3)(x-2)}$$

$$\begin{aligned}
&= \frac{2x^2 + 2x - 7 - x^2 - 2x + 3}{(x+3)(x-2)} \\
&= \frac{x^2 - 4}{(x+3)(x-2)} \\
&= \frac{x^2 - 2^2}{(x+3)(x-2)} \\
&= \frac{(x+2)(\cancel{x-2})}{(x+3)(\cancel{x-2})} \\
&= \frac{x+2}{x+3}
\end{aligned}$$

Q.9
$$= \frac{x^2 + x - 6}{x^2 - x - 6} \times \frac{x^2 - 4}{x^2 - 9}$$

Solution:
$$= \frac{x^2 + x - 6}{x^2 - x - 6} \times \frac{x^2 - 4}{x^2 - 9}$$

$$= \frac{x^2 + x - 6}{x^2 - x - 6} \times \frac{x^2 - 4}{x^2 - 9}$$

$$= \frac{x^2 + 3x - 2x - 6}{x^2 - 3x + 2x - 6} \times \frac{x^2 - 2^2}{x^2 - 3^2}$$

$$= \frac{x(x+3) - 2(x+3)}{2(x-3) + 2(x-3)} \times \frac{(x-2)(x+3)}{(x-3)(x+3)}$$

$$= \frac{(\cancel{x+3})(x-2)}{(x-3)(\cancel{x+2})} \times \frac{(x-2)(\cancel{x+2})}{(x-3)(\cancel{x+3})}$$

$$= \frac{(x-2)^2}{(x-3)^2}$$

Q.10
$$\frac{x^3 - 8}{x^2 - 4} \times \frac{x^2 + 6x + 8}{x^2 - 2x + 1}$$

Solution:
$$\frac{x^3 - 8}{x^2 - 4} \times \frac{x^2 + 6x + 8}{x^2 - 2x + 1}$$

$$= \frac{x^3 - 8}{x^2 - 4} \times \frac{x^2 + 6x + 8}{x^2 - 2x + 1}$$

$$\begin{aligned}
&= \frac{(x)^3 - (2)^3}{(x^2) - (2)^2} \times \frac{x^2 + 4x + 2x + 8}{x^2 - x - x + 1} \\
&= \frac{(x-2)(x^2 + 2x + 4)}{(x-2)(x+2)} \times \frac{x(x+4) + 2(x+4)}{x(x-1) - 1(x-1)} \\
&= \frac{x^2 + 2x + 4}{\cancel{(x+2)}} \times \frac{(x+4)\cancel{(x+2)}}{(x-1)(x-1)} \\
&= \frac{(x^2 + 2x + 4)(x+4)}{(x-1)^2}
\end{aligned}$$

Q.11 $\frac{x^4 - 8x}{2x^2 + 5x - 3} \times \frac{2x - 1}{x^2 + 2x + 4} \times \frac{x + 3}{x^2 - 2x}$

Solution: $\frac{x^4 - 8x}{2x^2 + 5x - 3} \times \frac{2x - 1}{x^2 + 2x + 4} \times \frac{x + 3}{x^2 - 2x}$

$$= \frac{x^4 - 8x}{2x^2 + 5x - 3} \times \frac{2x - 1}{x^2 + 2x + 4} \times \frac{x + 3}{x^2 - 2x}$$

$$= \frac{x(x^3 - 8)}{2x^2 + 6x - x - 3} \times \frac{2x - 1}{x^2 + 2x + 4} \times \frac{x + 3}{x(x - 2)}$$

$$= \frac{x[(x)^3 - (2)^3]}{2x(x + 3) - 1(x + 3)} \times \frac{2x - 1}{x^2 + 2x + 4} \times \frac{x + 3}{x(x - 2)}$$

$$= \frac{x\cancel{(x-2)}\cancel{(x^2 + 2x + 4)}}{\cancel{(2x-1)}\cancel{(x+3)}} \times \frac{\cancel{2x-1}}{\cancel{x^2 + 2x + 4}} \times \frac{\cancel{x+3}}{\cancel{x(x-2)}}$$

= 1 **Ans**

Q.12 $\frac{2y^2 + 7y - 4}{3y^2 - 13y + 4} \div \frac{4y^2 - 1}{6y^2 + y - 1}$

Solution: $\frac{2y^2 + 7y - 4}{3y^2 - 13y + 4} \div \frac{4y^2 - 1}{6y^2 + y - 1}$

$$\begin{aligned}
&= \frac{2y^2 + 7y - 4}{3y^2 - 13y + 4} \div \frac{4y^2 - 1}{6y^2 + y - 1} \\
&= \frac{2y^2 + 8y - 1y - 4}{3y^2 - 12y - y + 4} \div \frac{(2y)^2 - (1)^2}{6y^2 + 3y - 2y - 1} \\
&= \frac{2y(y+4) - 1(y+4)}{3y(y-4) - 1(y-4)} \div \frac{(2y-1)(2y+1)}{3y(2y+1) - 1(2y+1)} \\
&= \frac{(y+4)(2y-1)}{(3y-1)(y-4)} \div \frac{(2y-1)\cancel{(2y+1)}}{(3y-1)\cancel{(2y+1)}} \\
&= \frac{(y+4)\cancel{(2y-1)}}{\cancel{(3y-1)}(y-4)} \times \frac{\cancel{(3y-1)}}{\cancel{(2y-1)}} \\
&= \frac{y+4}{y-4}
\end{aligned}$$

Q.13

$$\left[\frac{x^2 + y^2}{x^2 - y^2} - \frac{x^2 - y^2}{x^2 + y^2} \right] \div \left[\frac{x+y}{x-y} - \frac{x-y}{x+y} \right]$$

Solution:
$$\left[\frac{x^2 + y^2}{x^2 - y^2} - \frac{x^2 - y^2}{x^2 + y^2} \right] \div \left[\frac{x+y}{x-y} - \frac{x-y}{x+y} \right]$$

$$= \left[\frac{x^2 + y^2}{x^2 - y^2} - \frac{x^2 - y^2}{x^2 + y^2} \right] \div \left[\frac{x+y}{x-y} - \frac{x-y}{x+y} \right]$$

$$= \left[\frac{(x^2 + y^2)^2 - (x^2 - y^2)^2}{(x^2 - y^2)(x^2 + y^2)} \right] \div \left[\frac{(x+y)^2 - (x-y)^2}{(x-y)(x+y)} \right]$$

$$= \left[\frac{(x^4 + 2x^2y^2 + y^4) - (x^4 - 2x^2y^2 + y^4)}{(x^2 - y^2)(x^2 + y^2)} \right] \div \left[\frac{(x^2 + 2xy + y^2) - (x^2 - 2xy + y^2)}{x^2 - y^2} \right]$$

$$= \left[\frac{\cancel{x^4} + 2x^2y^2 + \cancel{y^4} - \cancel{x^4} + 2x^2y^2 - \cancel{y^4}}{(x^2 - y^2)(x^2 + y^2)} \right] \div \left[\frac{\cancel{x^2} + 2xy + \cancel{y^2} - \cancel{x^2} + 2xy - \cancel{y^2}}{x^2 - y^2} \right]$$

$$= \left[\frac{4x^2y^2}{(x^2 - y^2)(x^2 + y^2)} \right] \div \left[\frac{4xy}{x^2 - y^2} \right]$$

$$= \frac{4x^2y^2}{(x^2 - y^2)(x^2 + y^2)} \times \frac{x^2 - y^2}{4xy}$$

$$= \frac{\cancel{4xy} \cdot xy}{(\cancel{x^2 - y^2})(x^2 + y^2)} \times \frac{\cancel{x^2 - y^2}}{\cancel{4xy}}$$

$$= \frac{xy}{x^2 + y^2} \text{ Ans}$$

Exercise 6.3

Q.1 Use factorization to find the square root of the following expression.

(i) $4x^2 - 12xy + 9y^2$

Solution: $4x^2 - 12xy + 9y^2$

$$4x^2 - 12xy + 9y^2 = 4x^2 - 6xy - 6xy + 9y^2$$

$$= 2x(2x - 3y) - 3y(3x - 3y)$$

$$= (2x - 3y)(2x - 3y)$$

$$4x^2 - 12xy + 9y^2 = (2x - 3y)^2$$

Taking square root on both side

$$\sqrt{4x^2 - 12xy + 9y^2} = \sqrt{[2x - 3y]^2}$$

$$= \pm(2x - 3y)$$

(ii) $x^2 - 1 + \frac{1}{4x^2}$

Solution: $x^2 - 1 + \frac{1}{4x^2}$

$$= (x)^2 - 2(x)\left[\frac{1}{2x}\right] + \left[\frac{1}{2x}\right]^2$$

$$= \left[x - \frac{1}{2x}\right]^2$$

Taking square root

$$\sqrt{x^2 - 1 + \frac{1}{4x^2}} = \sqrt{\left[x - \frac{1}{2x}\right]^2}$$

$$\sqrt{x^2 - 1 + \frac{1}{4x^2}} = \pm\left(x - \frac{1}{2x}\right)$$

(iii) $\frac{1}{16}x^2 - \frac{1}{12}xy + \frac{1}{36}y^2$

Solution: $\frac{1}{16}x^2 - \frac{1}{12}xy + \frac{1}{36}y^2$

$$= \left(\frac{1}{4}x\right)^2 - 2\left(\frac{1}{4}x\right)\left(\frac{1}{6}y\right) + \left(\frac{1}{6}y\right)^2$$

$$= \left(\frac{x}{4} - \frac{y}{6}\right)^2$$

Taking the square root

$$\begin{aligned}\sqrt{\frac{1}{16}x^2 - \frac{1}{12}xy + \frac{1}{36}y^2} &= \sqrt{\left(\frac{1}{4}x - \frac{1}{6}y\right)^2} \\ &= \pm\left(\frac{1}{4}x - \frac{1}{6}y\right) \\ &= \pm\left(\frac{x}{4} - \frac{y}{6}\right)\end{aligned}$$

(iv) $4(a+b)^2 - 12(a^2 + b^2) + 9(a-b)^2$

Solution: $4(a+b)^2 - 12(a^2 + b^2) + 9(a-b)^2$
 $= [2(a+b)^2] - 2[2(a+b)][3(a-b)] + [3(a-b)]^2$
 $= [2(a+b) - 3(a-b)]^2$

Taking square root

$$\begin{aligned}\sqrt{4(a+b)^2 - 12(a^2 + b^2) + 9(a-b)^2} &= \sqrt{[2(a+b) - 3(a-b)]^2} \\ &= \pm[2a + 2b - 3a + 3b] \\ &= \pm(5b - a)\end{aligned}$$

(v) $\frac{4x^6 - 12x^3y^3 + 9y^6}{9x^4 + 24x^2y^2 + 16y^4}$

Solution: $\frac{4x^6 - 12x^3y^3 + 9y^6}{9x^4 + 24x^2y^2 + 16y^4}$
 $= \frac{(2x^3)^2 - 2(2x^3)(3y^3) + (3y^3)^2}{(3x^2)^2 + 2(3x^2)(4y^2) + (4y^2)^2}$
 $= \frac{[2x^3 - 3y^3]^2}{[3x^2 + 4y^2]^2}$

Taking square root

$$\begin{aligned}&= \sqrt{\frac{4x^6 - 12x^3y^3 + 9y^6}{9x^4 + 24x^2y^2 + 16y^4}} \\ &= \pm\left(\frac{2x^3 - 3y^3}{3x^2 + 4y^2}\right)\end{aligned}$$

(vi) $\left(x + \frac{1}{x}\right)^2 - 4\left(x - \frac{1}{x}\right), (x \neq 0)$

Solution: $\left(x + \frac{1}{x}\right)^2 - 4\left(x - \frac{1}{x}\right), (x \neq 0)$

By adding and substituting 4

$$\begin{aligned}
&= x^2 + \frac{1}{x^2} + 2 - 4\left(x - \frac{1}{x}\right) \\
&= x^2 + \frac{1}{x^2} + 2 - 4\left(x - \frac{1}{x}\right) - 4 + 4 \\
&= x^2 + \frac{1}{x^2} - 2 - 4\left(x - \frac{1}{x}\right) + 4 \\
&= \left(x - \frac{1}{x}\right)^2 - 2\left(x - \frac{1}{x}\right)(2) + (2)^2 \\
&= \left[\left(x - \frac{1}{x}\right) - 2\right]^2
\end{aligned}$$

Taking square root

$$\begin{aligned}
\sqrt{\left(x - \frac{1}{x}\right)^2 - 4\left(x - \frac{1}{x}\right)} &= \sqrt{\left[x - \frac{1}{x} - 2\right]^2} \\
&= \pm\left(x - \frac{1}{x} - 2\right)
\end{aligned}$$

(vii) $\left(x^2 + \frac{1}{x^2}\right)^2 - 4\left(x + \frac{1}{x}\right)^2 + 12$

Solution: $\left(x^2 + \frac{1}{x^2}\right)^2 - 4\left(x + \frac{1}{x}\right)^2 + 12$

$$\begin{aligned}
&= \left[x^2 + \frac{1}{x^2}\right]^2 - 4\left[x^2 + \frac{1}{x^2} + 2\right] + 12 \\
&= \left[x^2 + \frac{1}{x^2}\right]^2 - 4x^2 - \frac{4}{x^2} - 8 + 12 \\
&= \left(x^2 + \frac{1}{x^2}\right)^2 - 4\left(x^2 + \frac{1}{x^2}\right) + 4 \\
&= \left[x^2 + \frac{1}{x^2}\right]^2 - 2\left[x^2 + \frac{1}{x^2}\right](2) + (2)^2 \\
&= \left[x^2 + \frac{1}{x^2} - 2\right]^2
\end{aligned}$$

Taking square root

$$\begin{aligned}
&= \sqrt{\left[x^2 + \frac{1}{x^2}\right]^2 - 4\left[x^2 + \frac{1}{x^2}\right] + 4} \\
&= \sqrt{\left[x^2 + \frac{1}{x^2} - 2\right]^2} \\
&= \pm\left(x^2 + \frac{1}{x^2} - 2\right)
\end{aligned}$$

(viii) $(x^2 + 3x + 2)(x^2 + 4x + 3)(x^2 + 5x + 6)$

Solution: $(x^2 + 3x + 2)(x^2 + 4x + 3)(x^2 + 5x + 6)$
 $= [x^2 + 2x + x + 2][x^2 + 3x + x + 3][x^2 + 3x + 2x + 6]$
 $= [x(x+2) + 1(x+2)][x(x+3) + 1(x+3)][x(x+3) + 2(x+3)]$
 $= (x+2)(x+1)(x+3)(x+1)(x+3)(x+2)$
 $= (x+2)^2(x+1)^2(x+3)^2$

Taking square root

$$= \sqrt{(x^2 + 3x + 2)(x^2 + 4x + 3)(x^2 + 5x + 6)}$$

$$= \sqrt{(x+2)^2(x+1)^2(x+3)^2}$$

$$= \pm(x+1)(x+2)(x+3) \text{ Ans}$$

(ix) $(x^2 + 8x + 7)(2x^2 - x - 3)(2x^2 + 11x - 21)$

Solution: $(x^2 + 8x + 7)(2x^2 - x - 3)(2x^2 + 11x - 21)$
 $= (x^2 + 7x + 1x + 7)(2x^2 - 3x + 2x - 3)(2x^2 + 14x - 3x - 21)$
 $= [(x(x+7) + 1(x+7))][x(2x-x) + 1(2x-3)][(2x(x+7) - 3(x+7))]$
 $= (x+7)(x+1)(2x-3)(x+1)(x+7)(2x-3)$
 $= (x+7)^2(x+1)^2(2x-3)^2$

Taking square root

$$= \sqrt{(x^2 + 8x + 7)(2x^2 - x - 3)(2x^2 + 11x - 21)}$$

$$= \sqrt{(x+7)^2(x+1)^2(2x-3)^2}$$

$$= \pm(x+1)(x+7)(2x-3) \text{ Ans}$$

Q.2 Use division method to find the square root of the following expression.

(i) $4x^2 + 12xy + 9y^2 + 16x + 24y + 16$

Solution: $4x^2 + 12xy + 9y^2 + 16x + 24y + 16$

$$\begin{array}{r} 2x+3y+4 \\ \hline 2x \overline{) 4x^2 + 12xy + 9y^2 + 16x + 24y + 16} \\ \underline{\pm 4x^2} \\ 4x+3y \overline{) 12xy + 9y^2 + 16x + 24y + 16} \\ \underline{\pm 12xy \pm 9y^2} \\ 4x+6y+4 \overline{) 16x + 24y + 16} \\ \underline{16x \pm 24y \pm 16} \\ 0 \end{array}$$

Square root = $\pm(2x+3y+4)$

(ii) $x^4 - 10x^3 + 37x^2 - 60x + 36$

Solution: $x^4 - 10x^3 + 37x^2 - 60x + 36$

$$\begin{array}{r}
 \overline{) x^4 - 10x^3 + 37x^2 - 60x + 36} \\
 \underline{\pm x^4} \\
 2x^2 - 5x \overline{) 10x^3 + 37x^2 - 60x + 36} \\
 \underline{\pm 10x^3 \pm 25x^2} \\
 2x^2 - 10x + 6 \overline{) 12x^2 + 60x + 36} \\
 \underline{\pm 12x^2 \pm 60x \pm 36} \\
 \\
 \times
 \end{array}$$

Square root $= \pm(x^2 - 5x + 6)$

(iii) $9x^4 - 6x^3 + 7x^2 - 2x + 1$

Solution: $9x^4 - 6x^3 + 7x^2 - 2x + 1$

$$\begin{array}{r}
 \overline{) 9x^4 - 6x^3 + 7x^2 - 2x + 1} \\
 \underline{\pm 9x^4} \\
 6x^2 - x \overline{) 6x^3 + 7x^2 - 2x + 1} \\
 \underline{\pm 6x^3 \pm 7x^2} \\
 6x^2 - 2x + 1 \overline{) 6x^2 - 2x + 1} \\
 \underline{\pm 6x^2 \pm 2x \pm 1} \\
 \\
 \times
 \end{array}$$

Square root $\pm(= 3x^2 - x + 1)$

(iv) $4 + 25x^2 + 7x^2 - 2x + 1$

Solution: $4 + 25x^2 - 12x - 24x^3 + 16x^4$

$$\begin{array}{r}
 \overline{4x^2 - 3x + 2} \\
 4x^2 \overline{) \cancel{16x^4} - 24x^3 + 25x^2 - 12x + 4} \\
 \underline{ \pm 16x^4} \\
 8x^2 - 3x \overline{) \cancel{-24x^3} + 25x^2 - 12x + 4} \\
 \underline{ \pm 24x^3 \pm 9x^2} \\
 8x^2 - 6x + 2 \overline{) \cancel{16x^2} - \cancel{12x} + \cancel{4}} \\
 \underline{ \pm 16x^2 \pm 12x \pm 4} \\
 \phantom{) \phantom{\cancel{16x^2} - \cancel{12x} + \cancel{4}}} \\
 \phantom{) \phantom{\cancel{16x^2} - \cancel{12x} + \cancel{4}}} \times
 \end{array}$$

Square root = $\pm(4x^2 - 3x + 2)$

(v) $\frac{x^2}{y^2} - \frac{10x}{y} + 27 - \frac{10y}{x} + \frac{y^2}{x^2}, (x \neq 0, y \neq 0)$

Solution: $\frac{x^2}{y^2} - \frac{10x}{y} + 27 - \frac{10y}{x} + \frac{y^2}{x^2}, (x \neq 0, y \neq 0)$

$$\begin{array}{r}
 \phantom{\frac{x}{y}} \overline{\frac{x}{y} - 5 + \frac{y}{x}} \\
 \frac{x}{y} \overline{) \frac{x^2}{y^2} - \frac{10x}{y} + 27 - \frac{10y}{x} + \frac{y^2}{x^2}} \\
 \underline{\phantom{\frac{x}{y}} \pm \frac{x^2}{y^2}} \phantom{- \frac{10x}{y} + 27 - \frac{10y}{x} + \frac{y^2}{x^2}} \\
 \phantom{\frac{x}{y}} \overline{\frac{2x}{y} - 5} \overline{) \cancel{-\frac{10x}{y}} + 27 - \frac{10y}{x} + \frac{y^2}{x^2}} \\
 \underline{\phantom{\frac{x}{y}} \phantom{\overline{\frac{2x}{y} - 5}} \pm \frac{x^2}{y^2} \pm 25} \phantom{- \frac{10y}{x} + \frac{y^2}{x^2}} \\
 \phantom{\frac{x}{y}} \phantom{\overline{\frac{2x}{y} - 5}} \phantom{) \cancel{-\frac{10x}{y}} + 27 - \frac{10y}{x} + \frac{y^2}{x^2}} \\
 \phantom{\frac{x}{y}} \phantom{\overline{\frac{2x}{y} - 5}} \phantom{) \cancel{-\frac{10x}{y}} + 27 - \frac{10y}{x} + \frac{y^2}{x^2}} \times
 \end{array}$$

Square root = $\pm\left(\frac{x}{y} - 5 + \frac{y}{x}\right)$

Q.3 Find the value of k for which the following expressions will become a perfect square.

(i) $4x^4 - 12x^3 + 37x^2 - 42x + k$

Solution: $4x^4 - 12x^3 + 37x^2 - 42x + k$

$$\begin{array}{r}
 \overline{2x^2 - 3x + 7} \\
 2x^2 \overline{) 4x^4 - 12x^3 + 37x^2 - 42x + k} \\
 \underline{\pm 4x^4} \\
 4x^2 - 3x \overline{) -12x^3 + 37x^2 - 42x + k} \\
 \underline{\pm 12x^3 \pm 9x^2} \\
 4x^2 - 6x + 7 \overline{) 28x^2 - 42x + k} \\
 \underline{\pm 28x^2 \pm 42x \pm 49} \\
 k - 49
 \end{array}$$

In the case of perfect square remainder is always is equal to zero so

$$k - 49 = 0$$

$$k = 49$$

(ii) $x^4 - 4x^3 + 10x^2 - kx + 9$

Solution: $x^4 - 4x^3 + 10x^2 - kx + 9$

$$\begin{array}{r}
 \overline{x^2 - 2x + 3} \\
 x^2 \overline{) x^4 - 4x^3 + 10x^2 - kx + 9} \\
 \underline{\pm x^4} \\
 2x^2 - 2x \overline{) -4x^3 + 10x^2 - kx + 9} \\
 \underline{\pm 4x^3 \pm 4x^2} \\
 2x^2 - 4x + 3 \overline{) 6x^2 - kx + 9} \\
 \underline{-6x^2 \mp 12x \pm 9}
 \end{array}$$

$$-kx + 12x = 0$$

In the case of square root remainder is always equal to zero

$$-x(k - 12) = 0$$

$$k - 12 = \frac{0}{-x}$$

$$k - 12 = 0$$

$$k = 12$$

Q.5 To make the expression $9x^4 - 12x^3 + 22x^2 - 13x + 12$ a perfect square

Solution: $9x^4 - 12x^3 + 22x^2 - 13x + 12$

$$\begin{array}{r} 3x^2 - 2x + 3 \\ \hline 3x^2 \overline{) 9x^4 - 12x^3 + 22x^2 - 13x + 12} \\ \underline{\pm 9x^4} \\ 6x^2 - 2x \overline{) -12x^3 + 22x^2 - 13x + 12} \\ \underline{\pm 12x^3 \pm 4x^2} \\ 6x^2 - 4x + 3 \overline{) 18x^2 - 13x + 12} \\ \underline{\pm 18x^2 \mp 12x \pm 9} \\ -x + 3 \end{array}$$

- (i) $+x - 3$ is to be added
- (ii) $-x + 3$ is to be subtract from it
- (iii) $-x + 3 = 0$
 $x = 3$

Review Exercise 6

Q.1 Choose the correct answer.

- (i) **H.C.F of $p^3q - pq^3$ and $p^5q^2 - pq^5$ is _____**
(a) $pq(p^2 - q^2)$ (b) $pq(p - q)$
(c) $p^2q^2(p - q)$ (d) $pq(p^3 - q^3)$
- (ii) **H.C.F of $5x^2y^2$ and $20x^3y^3$ is _____**
(a) $5x^2y^2$ (b) $20x^3y^3$
(c) $100x^5y^5$ (d) $5xy$
- (iii) **H.C.F of $x - 2$ and $x^2 + x - 6$ _____**
(a) $x^2 + x - 6$ (b) $x + 3$
(c) $x - 2$ (d) $x + 2$
- (iv) **H.C.F of $a^3 + b^3$ and $a^2 - ab + b^2$ _____**
(a) $a + b$ (b) $a^2 - ab + b^2$
(c) $(a - b)^2$ (d) $a^2 + b^2$
- (v) **H.C.F of $x^2 - 5x + 6$ and $x^2 - x - 6$ is _____**
(a) $x - 3$ (b) $x + 2$
(c) $x^2 - 4$ (d) $x - 2$
- (vi) **H.C.F of $a^2 - b^2$ and $a^3 - b^3$ is _____**
(a) $a - b$ (b) $a + b$
(c) $a^2 + ab + b^2$ (d) $a^2 - ab + b^2$
- (vii) **H.C.F of $x^2 + 3x + 2, x^2 + 4x + 3$ and $x^2 + 5x + 4$ is _____**
(a) $x + 1$ (b) $(x + 1)(x + 2)$
(c) $x + 3$ (d) $(x + 4)(x + 1)$
- (viii) **L.C.M of $15x^2, 45xy$ and $30xyz$ is _____**
(a) $90xyz$ (b) $90x^2yz$
(c) $15xyz$ (d) $15x^2yz$
- (ix) **L.C.M of $a^2 + b^2$ and $a^4 - b^4$ is _____**
(a) $a^2 + b^2$ (b) $a^2 - b^2$
(c) $a^4 - b^4$ (d) $a - b$
- (x) **The product of two algebraic expression is equal to the _____ of their H.C.F and L.C.M**
(a) Sum (b) Difference
(c) Product (d) Quotient
- (xi) **Simplify $\frac{a}{9a^2 - b^2} + \frac{1}{3a - b}$ is _____**

(a) $\frac{4a}{9a^2 - b^2}$

(b) $\frac{4a - b}{9a^2 - b^2}$

(c) $\frac{4a + b}{9a^2 - b^2}$

(d) $\frac{b}{9a^2 - b^2}$

(xii) Simplify $\frac{a^2 + 5a - 14}{a^2 - 3a - 18} \times \frac{a + 3}{a - 2} =$ _____

(a) $\frac{a + 7}{a - 6}$

(b) $\frac{a + 7}{a - 2}$

(c) $\frac{a + 3}{a - 6}$

(d) $\frac{a - 2}{a + 3}$

(xiii) Simplify the $\frac{a^3 - b^3}{a^4 - b^4} \div \frac{a^2 + ab + b^2}{a^2 + b^2} =$ _____

(a) $\frac{1}{a + b}$

(b) $\frac{1}{a - b}$

(c) $\frac{a - b}{a^2 + b^2}$

(d) $\frac{a + b}{a^2 + b^2}$

(xiv) Simplify $\left(\frac{2x + y}{x + y} - 1\right) \div \left(1 - \frac{x}{x + y}\right) =$ _____

(a) $\frac{x}{x + y}$

(b) $\frac{y}{x + y}$

(c) $\frac{y}{x}$

(d) $\frac{x}{y}$

(xv) The square root of $a^2 - 2a + 1$ is _____

(a) $\pm(a + 1)$

(b) $\pm(a - 1)$

(c) $a - 1$

(d) $a + 1$

(xvi) What should be added to complete the square of $x^4 + 64$? _____

(a) $8x^2$

(b) $-8x^2$

(c) $16x^2$

(d) $4x^2$

(xvii) The square root to $x^4 + \frac{1}{x^4} + 2$ is _____

(a) $\pm\left(x + \frac{1}{x}\right)$

(b) $\pm\left(x^2 + \frac{1}{x^2}\right)$

(c) $\pm\left(x - \frac{1}{x}\right)$

(d) $\pm\left(x^2 - \frac{1}{x^2}\right)$

ANSWER KEYS

1	b	5	a	9	c	13	a	17	b
2	a	6	a	10	c	14	d		
3	c	7	a	11	c	15	b		
4	b	8	b	12	a	16	c		

Q.2 Find the H.C.F of the following by factorization.

$$8x^4 - 128, 12x^3 - 96$$

Solution:

$$\begin{aligned} 8x^4 - 128 &= 8(x^4 - 16) = 8[(x)^2 - (4)^2] \\ &= 2 \times 2 \times 2(x^2 + 4)(x^2 - 4) \\ &= 2 \times 2 \times 2(x^2 + 4)(x + 2)(x - 2) \end{aligned}$$

$$\begin{aligned} 12x^3 - 96 &= 12(x^3 - 8) \\ &= (12(x^3 - 2^3)) \\ &= 12(x - 2)(x^2 + 2x + 4) \end{aligned}$$

$$2 \times 2 \times 3(x - 2)(x^2 + 2x + 4)$$

$$\mathbf{H.C.F} = 2 \times 2(x - 2)$$

$$= 4(x - 2)$$

Q.3 Find the H.C.F of the following by division method $y^3 + 3y^2 - 3y - 9, 3y^2 - 8y - 24$.

Solution: $y^3 + 3y^2 - 3y - 9,$

$$= y^3 + 3y^2 - 3y - 9$$

$$\begin{array}{r} y^3 + 3y^2 - 8y - 24 \overline{) y^3 + 3y^2 - 3y - 9} \\ \underline{\pm y^3 \pm 3y^2 \pm 8y \pm 24} \\ 5y + 15 \\ 5(y + 3) \end{array}$$

$$\begin{array}{r} y^2 - 8 \overline{) y^3 + 3y^2 - 8y - 24} \\ \underline{\pm y^3 \pm 3y^2} \\ -8y - 24 \\ \underline{\pm 8y \pm 24} \\ \times \end{array}$$

$$\mathbf{H.C.F} = (y + 3)$$

Q.4 Find the L.C.M of the following by factorization.

$$12x^2 - 75, 6x^2 - 13x - 5, 4x^2 - 20x + 25$$

Solution:

$$\begin{aligned} 12x^2 - 75 &= 3(4x^2 - 25) \\ &= 3[(2x)^2 - (5)^2] \\ &= 3(2x - 5)(2x + 5) \end{aligned}$$

$$\begin{aligned} 6x^2 - 15x + 2x - 5 &= 3x(2x - 5) + 1(2x - 5) \\ &= (2x - 5)(3x + 1) \end{aligned}$$

$$\begin{aligned} 4x^2 - 20x + 25 &= 4x^2 - 10x - 10x + 25 \\ &= 2x(2x - 5) - 5(2x - 5) \\ &= (2x - 5)(2x - 5) \end{aligned}$$

Common factor = $(2x-5)$

Non common factor = $3(3x+1)(2x-5)2x+5)$

L.C.M = common factor \times non common factor

$$\text{L.C.M} = (2x-5)3(3x+1)(2x+5)(2x-5)$$

$$\text{L.C.M} = 3(2x+5)(2x-5)^2(3x+1)$$

Q.5 If H.C.F of $x^4 + 3x^3 + 5x^2 + 26x + 56$ and $x^4 + 2x^3 - 4x^2 - x + 28$ is $x^2 + 5x + 7$ find their L.C.M.

Solution: $p(x) = x^4 + 3x^3 + 5x^2 + 26x + 56$ and $q(x) = x^4 + 2x^3 - 4x^2 - x + 28$

$HCF = x^2 + 5x + 7$, $LCM = ?$

$$\text{L.C.M} = \frac{P(x) \times q(x)}{H.C.F}$$

$$\text{L.C.M} = \frac{(x^4 + 3x^3 + 5x^2 + 26x + 56) \times (x^4 + 2x^3 - 4x^2 - x + 28)}{(x^2 + 5x + 7)}$$

$$\begin{array}{r} x^2 + 5x + 7 \overline{) x^4 + 2x^3 - 4x^2 - x + 28} \\ \underline{\pm x^4 \pm 5x^3 \pm 7x^2} \\ -3x^3 - 11x^2 - x + 28 \end{array}$$

$$\begin{array}{r} \underline{\mp 3x^3 \mp 15x^2 \mp 21x} \\ +4x^2 + 20x + 28 \\ \underline{\pm 4x^2 \pm 20x \pm 28} \end{array}$$

$$x^2 - 3x + 4$$

$$L.C.M = \frac{(x^4 + 3x^3 + 5x^2 + 26x + 56) \cancel{(x^4 + 2x^3 - 4x^2 - x + 28)}}{\cancel{(x^2 + 5x + 7)}}$$

$$L.C.M = (x^4 + 3x^3 + 5x^2 + 26x + 56)(x^2 - 3x + 4)$$

Q.6 Simplify:

Solution:

$$(i) \quad \frac{3}{x^3 + x^2 + x + 1} - \frac{3}{x^3 - x^2 + x - 1}$$

$$\begin{aligned} \text{Solution: } & \frac{3}{x^3 + x^2 + x + 1} - \frac{3}{x^3 - x^2 + x - 1} \\ &= \frac{3}{x^2(x+1)+1(x+1)} - \frac{3}{x^2(x-1)+1(x-1)} \\ &= \frac{3}{(x^2+1)(x+1)} - \frac{3}{(x-1)(x^2+1)} \\ &= \frac{3(x-1) - 3(x+1)}{(x+1)(x-1)(x^2+1)} \\ &= \frac{\cancel{3x} - 3 - \cancel{3x} - 3}{(x-1)(x-1)(x^2+1)} \\ &= \frac{-6}{(x+1)(x-1)(x^2+1)} = \frac{-6}{(x^2-1)(x^2+1)} \\ &= \frac{-6}{(x^4-1)} \\ &= \frac{6}{1-x^4} \end{aligned}$$

$$(ii) \quad \frac{a+b}{a^2-b^2} \div \frac{a^2-ab}{a^2-2ab+b^2}$$

$$\begin{aligned} \text{Solution: } & \frac{a+b}{a^2-b^2} \div \frac{a^2-ab}{a^2-2ab+b^2} \\ &= \frac{a+b}{a^2-b^2} \times \frac{a^2-2ab+b^2}{a^2-ab} \\ &= \frac{\cancel{a+b}}{(a-b)(\cancel{a+b})} \times \frac{(a-b)^2}{a(a-b)} \end{aligned}$$

$$\begin{aligned} &= \frac{(\cancel{a-b})^2}{a(\cancel{a-b})^2} \\ &= \frac{1}{a} \end{aligned}$$

Q.7 Find the square root by using factorization. $\left(x^2 + \frac{1}{x^2}\right) + 10\left(x + \frac{1}{x}\right) + 27 \quad (x \neq 0).$

$$\begin{aligned} \text{Solution: } & \left(x^2 + \frac{1}{x^2}\right) + 10\left(x + \frac{1}{x}\right) + 27 \\ &= \left(x^2 + \frac{1}{x^2}\right) + 10\left(x + \frac{1}{x}\right) + 25 + 2 \\ &= x^2 + \frac{1}{x^2} + 2 + 10\left(x + \frac{1}{x}\right) + 25 \\ &= \left(x + \frac{1}{x}\right)^2 + 2\left(x + \frac{1}{x}\right) \times 5 + (5)^2 \\ &= \left[x + \frac{1}{x} + 5\right]^2 \end{aligned}$$

Taking the square root

$$\begin{aligned} \sqrt{\left(x^2 + \frac{1}{x^2}\right) + 10\left(x + \frac{1}{x}\right) + 27} &= \sqrt{\left[x + \frac{1}{x} + 5\right]^2} \\ &= \pm \left(x + \frac{1}{x} + 5\right) \end{aligned}$$

Q.8 Find the square roots by using division method. $\frac{4x^2}{y^2} + \frac{20x}{y} + 13 - \frac{30y}{x} + \frac{9y^2}{x^2}$

Solution:

$$\begin{array}{r} \frac{2x}{y} + 5 - \frac{3y}{x} \\ \hline \frac{2x}{y} \left(\frac{4x^2}{y^2} + \frac{20x}{y} + 13 - \frac{30y}{x} + \frac{9y^2}{x^2} \right) \\ \hline \pm \frac{4x^2}{y^2} \\ \hline \frac{4x}{y} + 5 \left(\frac{20x}{y} + 13 - \frac{30y}{x} + \frac{9y^2}{x^2} \right) \\ \hline \pm \frac{20x}{y} \pm 25 \end{array}$$

$$\frac{4x}{y} + 10 - \frac{3y}{x} \left(-12 - \frac{30y}{x} + \frac{9y^2}{x^2} \right)$$

$$\pm 12 \mp \frac{30y}{x} \pm \frac{9y^2}{x^2}$$

Square root = $\pm \left[\frac{2x}{y} + 5 - \frac{3y}{x} \right]$

Unit 6: Algebraic Manipulation

Overview

Highest Common Factor:

If two or more algebraic expressions are given then their common factor of highest power is called the H.C.F. of the expression.

Least Common Multiple(L.C.M):

The Least common Multiple (L.C.M) is the product of common factors together with non-common factors of the given expressions.