

Exercise 14.1

Q.1 In $\triangle ABC$
 $\overline{DE} \parallel \overline{BC}$

(i) If $\overline{AD} = 1.5\text{cm}$ $\overline{BD} = 3\text{cm}$

$\overline{AE} = 1.3\text{cm}$, then find \overline{CE}

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

By substituting the values of \overline{AD} , \overline{BD} and \overline{AE}

So

$$\frac{1.5}{3} = \frac{1.3}{\overline{EC}}$$

$$\overline{EC}(1.5) = 1.3 \times 3$$

$$\overline{EC} = \frac{1.3 \times 3}{1.5}$$

$$\overline{EC} = \frac{3.9}{1.5}$$

$$\overline{EC} = 2.6\text{cm}$$

(ii) If $\overline{AD} = 2.4\text{cm}$ $\overline{AE} = 3.2\text{cm}$

$\overline{EC} = 4.8\text{cm}$ find \overline{AB}

$$\frac{\overline{AD}}{\overline{AB}} = \frac{\overline{AE}}{\overline{AC}}$$

$$\overline{AC} = \overline{AE} + \overline{EC}$$

$$\overline{AC} = 3.2 + 4.8$$

$$\overline{AC} = 8\text{cm}$$

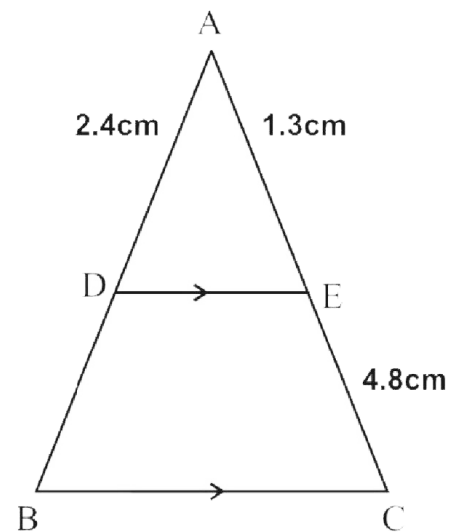
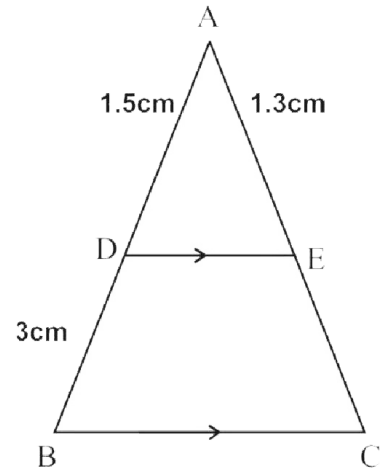
$$\therefore \frac{\overline{AD}}{\overline{AB}} = \frac{\overline{AE}}{\overline{AC}}$$

$$\frac{2.4}{\overline{AB}} = \frac{3.2}{8}$$

$$2.4 \times 8 = (3.2)\overline{AB}$$

$$\frac{19.2}{3.2} = \overline{AB}$$

$$\overline{AB} = 6\text{cm}$$



(iii) If $\frac{\overline{AD}}{\overline{BD}} = \frac{3}{5} \overline{AC} = 4.8\text{cm}$ find \overline{AE}

$$\overline{AC} = \overline{AE} + \overline{EC}$$

$$\overline{AC} = \overline{EC} + \overline{AE}$$

$$\overline{AE} = 4.8 - \overline{EC}$$

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AC} - \overline{EC}}{\overline{EC}}$$

$$\frac{3}{5} = \frac{4.8 - \overline{EC}}{\overline{EC}}$$

$$3(\overline{EC}) = 5(4.8 - \overline{EC})$$

$$3(\overline{EC}) = 24 - 5(\overline{EC})$$

$$3(\overline{EC}) + 5(\overline{EC}) = 24$$

$$8(\overline{EC}) = 24$$

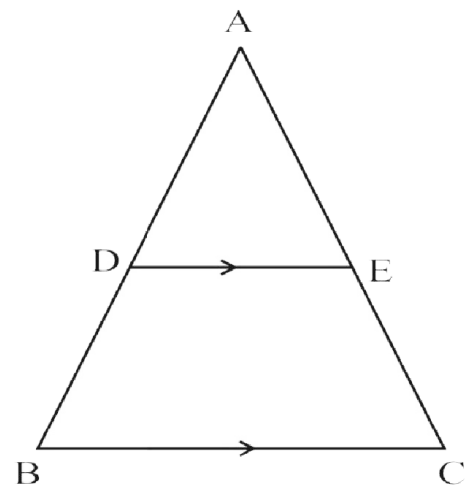
$$(\overline{EC}) = \frac{24}{8}$$

$$\overline{EC} = 3\text{cm}$$

$$\overline{AE} = \overline{AC} - \overline{EC}$$

$$= 4.8 - 3$$

$$= 1.8\text{cm}$$



(iv) If $\overline{AD} = 2.4\text{cm}$, $\overline{AE} = 3.2\text{cm}$, $\overline{DE} = 2\text{cm}$, $\overline{BC} = 5\text{cm}$. Find \overline{AB} , \overline{DB} , \overline{AC} , \overline{CE} .

$$\frac{\overline{AD}}{\overline{AB}} = \frac{\overline{AE}}{\overline{AC}} = \frac{\overline{DE}}{\overline{BC}}$$

$$\frac{2.4}{\overline{AB}} = \frac{3.2}{\overline{AC}} = \frac{2}{5}$$

$$\frac{2.4}{\overline{AB}} = \frac{2}{5}$$

$$(2.4)5 = 2(\overline{AB})$$

$$\frac{12.0}{2} = \overline{AB}$$

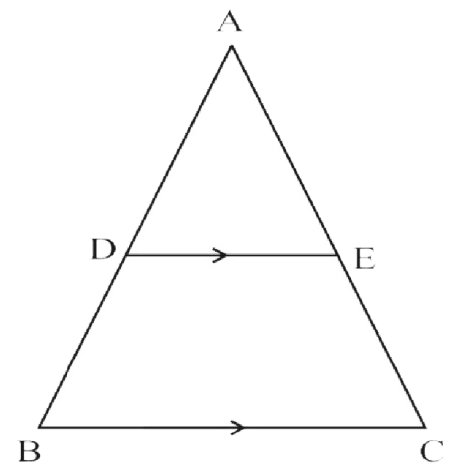
$$\overline{AB} = 6\text{cm}$$

$$\frac{3.2}{\overline{AC}} = \frac{2}{5}$$

$$16.0 = 2(\overline{AC})$$

$$\frac{16}{2} = \overline{AC}$$

$$\overline{AC} = 8\text{cm}$$



$$\begin{aligned} \overline{DB} &= \overline{AB} - \overline{AD} \\ \overline{DB} &= 6 - 2.4 \\ \overline{DB} &= 3.6 \text{ cm} \\ \frac{\overline{AD}}{\overline{AB}} &= \frac{\overline{AE}}{\overline{AC}} \\ \frac{2.4}{6} &= \frac{\overline{AE}}{8} \\ \overline{AE} &= \frac{2.4}{6} \times 8 \\ \overline{AE} &= \frac{19.2}{6} \\ \overline{AE} &= 3.2 \text{ cm} \\ \overline{CE} &= \overline{AC} - \overline{AE} \\ \overline{CE} &= 8 - 3.2 \\ \overline{CE} &= 4.8 \text{ cm} \end{aligned}$$

If $\overline{AD} = 4x - 3$ $\overline{AE} = 8x - 7$

$\overline{BD} = 3x - 1$ and $\overline{CE} = 5x - 3$ Find the value of x

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

By putting the value of \overline{AD} , \overline{AE} , \overline{BD} and \overline{CE}

$$\frac{4x - 3}{3x - 1} = \frac{8x - 7}{5x - 3}$$

By cross multiplying

$$\begin{aligned} (4x - 3)(5x - 3) &= (8x - 7)(3x - 1) \\ 20x^2 - 12x - 15x + 9 &= 24x^2 - 8x - 21x + 7 \\ 20x^2 - 27x + 9 &= 24x^2 - 29x + 7 \\ 0 &= 24x^2 - 20x^2 - 29x + 27x + 7 - 9 \\ 4x^2 - 2x - 2 &= 0 \\ 2(2x^2 - x - 1) &= 0 \end{aligned}$$

$$2x^2 - 2x + 1x - 1 = \frac{0}{2}$$

$$2x(x - 1) + 1(x - 1) = 0$$

$$(x - 1)(2x + 1) = 0$$

$$x - 1 = 0$$

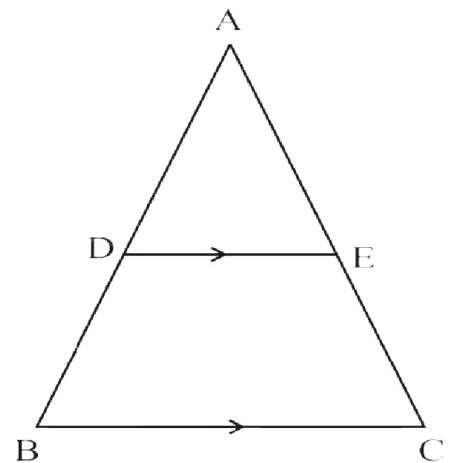
$$x = 1$$

$$2x + 1 = 0$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

Distance is not taken in negative it is always in positive so the value of $x = 1$.



Q.2 In $\triangle ABC$ is an isosceles triangle $\angle A$ is vertex angle and \overline{DE} intersects the sides \overline{AB} and \overline{AC} as shown in the figure so that

$$m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$$

Prove that $\triangle ADE$ is also an isosceles triangle.

Given:

$\triangle ABC$ is an isosceles triangle, $\angle A$ is vertex and \overline{DE} intersects the sides \overline{AB} and \overline{AC} .

$$\frac{m\overline{AD}}{m\overline{BD}} = \frac{m\overline{AE}}{m\overline{EC}}$$

To Prove

$$m\overline{AD} = m\overline{AE}$$

Proof

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

$$\text{Or } \frac{\overline{BD}}{\overline{AD}} = \frac{\overline{EC}}{\overline{AE}}$$

$$\text{Or } \frac{\overline{AD} + \overline{BD}}{\overline{AD}} = \frac{\overline{AE} + \overline{EC}}{\overline{AE}}$$

As we know

$$\overline{AB} = \overline{AD} + \overline{BD}$$

$$\overline{AC} = \overline{AE} + \overline{EC}$$

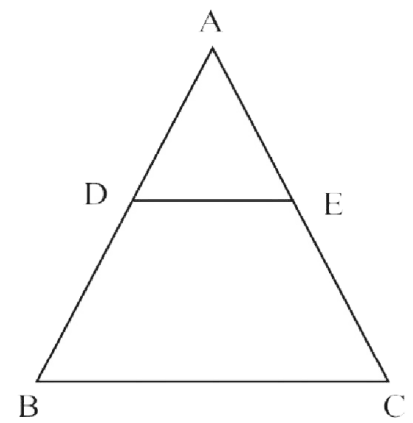
$$\frac{\overline{AB}}{\overline{AD}} = \frac{\overline{AC}}{\overline{AE}}$$

From this

$$\frac{\overline{AB}}{\overline{AD}} = \frac{\overline{AC}}{\overline{AE}}$$

$$\overline{AD} = \overline{AE}$$

$$\overline{AB} = \overline{AC} \text{ (Given)}$$



Q.3 In an equilateral triangle ABC shown in the figure $m\overline{AE}:m\overline{AC} = m\overline{AD}:m\overline{AB}$ find all the three angles of $\triangle ADE$ and name it also.

Given

$\triangle ABC$ is equilateral triangle

To prove

To find the angles of $\triangle ADE$

Solution:

$$\frac{m\overline{AE}}{m\overline{AC}} = \frac{m\overline{AD}}{m\overline{AB}}$$

All angles are equal as it is an equilateral triangle which are equal to 60° each

$$\angle A = \angle B = \angle C$$

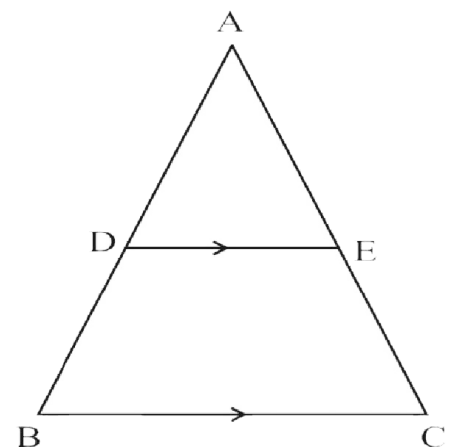
$$m\overline{BC} \parallel m\overline{DE}$$

$$\angle ADE = \angle ABC = 60^\circ$$

$$\angle AED = \angle ACB = 60^\circ$$

$$\angle A = 60^\circ$$

$\triangle ADE$ is an equilateral triangle



Q.4 Prove that line segment drawn through the midpoint of one side of a triangle and parallel to another side bisect the third side

Given

$$\overline{AD} = \overline{BD}$$

$$\overline{DE} \parallel \overline{BC}$$

To Prove

$$\overline{AE} = \overline{EC}$$

In $\triangle ABC$

$$\overline{DE} \parallel \overline{BC}$$

In theorem it is already discussed that

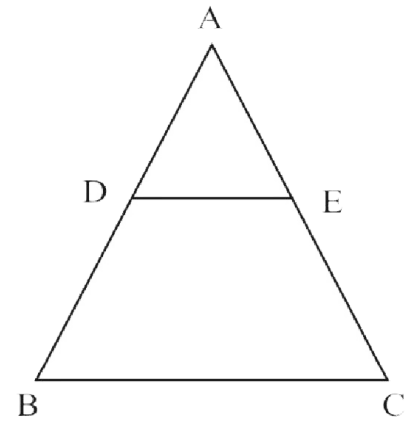
$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

As we know $\overline{AD} = \overline{BD}$ or $\overline{BD} = \overline{AD}$

$$\frac{\overline{AD}}{\overline{AD}} = \frac{\overline{AE}}{\overline{EC}}$$

$$1 = \frac{\overline{AE}}{\overline{EC}}$$

$$\overline{EC} = \overline{AE}$$



Q.5 Prove that the line segment joining the midpoint of any two sides of a triangle is parallel to the third side

Given

$\triangle ABC$ the midpoint of \overline{AB} and \overline{AC} are L and M respectively

To Prove

$$\overline{LM} \parallel \overline{BC} \text{ and } m\overline{LM} = \frac{1}{2} \overline{BC}$$

Construction

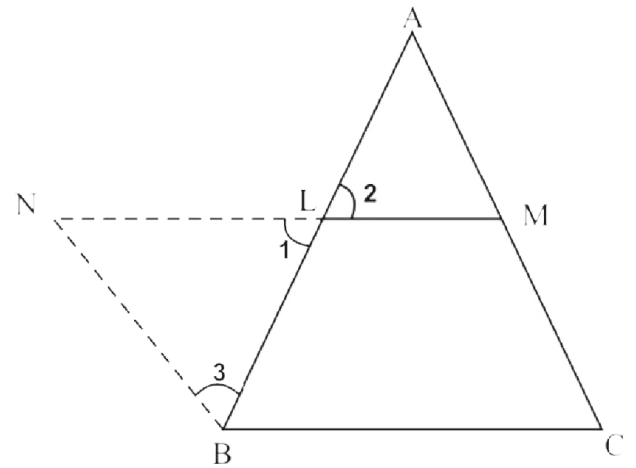
Join M to L and produce \overline{ML} to N such that

$$\overline{ML} \cong \overline{LN}$$

Join N to B and in the figure name the angles

$\angle 1$, $\angle 2$, and $\angle 3$

Proof



Statements	Reasons
$\triangle BLN \leftrightarrow \triangle ALM$	
$\overline{BL} \cong \overline{AL}$	Given
$\angle 2 = \angle 1$ or $\angle 1 = \angle 2$	Vertical angles
$\overline{NL} = \overline{ML}$	Construction
$\therefore \triangle BLN \cong \triangle ALM$	Corresponding angle of congruent triangles
$\therefore \angle A = \angle 3$	Given
And $\overline{NB} \cong \overline{AM}$	
$\overline{NB} \parallel \overline{AM}$	

$\overline{ML} = \overline{AM}$ $\overline{NB} \cong \overline{ML}$ \overline{BCMN} is parallelogram $\therefore \overline{BC} \parallel \overline{LM}$ or $\overline{BC} \parallel \overline{NL}$ $\overline{BC} \cong \overline{NM}$ $m\angle LM = \frac{1}{2} m\angle NM$ Hence $m\angle LM = \frac{1}{2} m\angle BC$	Given (Opposite side of parallelogram BCMN) (Opposite side of parallelogram)
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Theorem 14.1.3

The internal bisector of an angle of a triangle divides the sides opposite to it in the ratio of the lengths of the sides containing the angle.

Given

In $\triangle ABC$ internal angle bisector of $\angle A$ meets \overline{CB} at the points D.

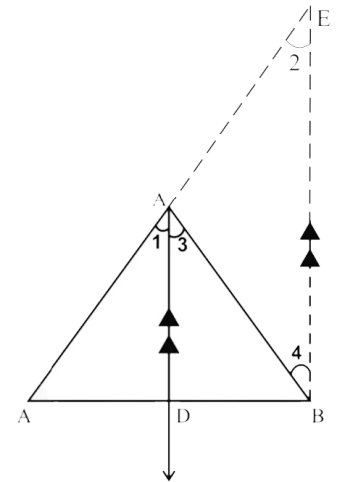
To prove

$$m\overline{BD} : m\overline{DC} = m\overline{AB} : m\overline{AC}$$

Construction

Draw a line segment $\overline{BE} \parallel \overline{DA}$ to meet \overline{CA} Produced at E

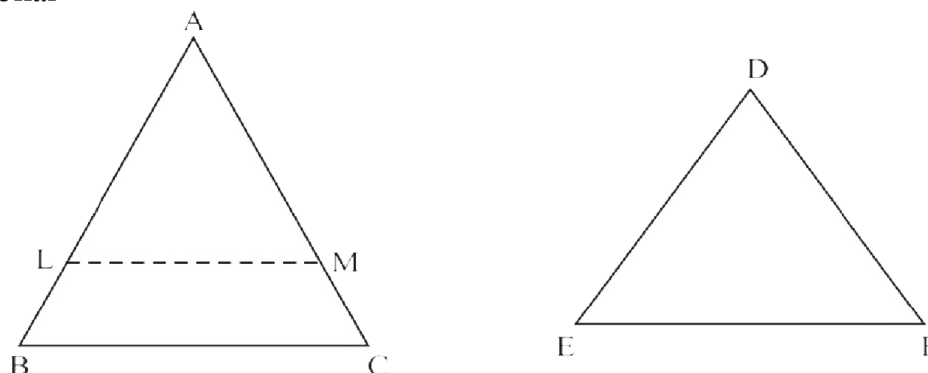
Proof



Statements	Reasons
$\therefore \overline{AD} \parallel \overline{EB}$ and \overline{EC} intersect them	Construction
$m\angle 1 = m\angle 2 \dots \dots \dots (i)$	Corresponding angles
Again $\overline{AD} \parallel \overline{EB}$ and \overline{AB} intersects them	
$\therefore m\angle 3 = m\angle 4 \dots \dots \dots (ii)$	Alternate angles
But $m\angle 1 = m\angle 3$	Given
$\therefore m\angle 2 = m\angle 4$	From (i) and (ii)
And $\overline{AB} \cong \overline{AE}$ or $\overline{AE} \cong \overline{AB}$	In a Δ , the sides opposite to congruent angles are also congruent
Now $\overline{AD} \parallel \overline{EB}$	Construction
$\therefore \frac{m\overline{BD}}{m\overline{DC}} = \frac{m\overline{EA}}{m\overline{AC}}$	A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.
or $\frac{m\overline{BD}}{m\overline{DC}} = \frac{m\overline{AB}}{m\overline{AC}}$	$m\overline{EA} = m\overline{AB}$ (proved)
Thus $m\overline{BD} : m\overline{DC} = m\overline{AB} : \overline{AC}$	

Theorem 14.1.4

If two triangles are similar, then the measures of their corresponding sides are proportional



Given

$\Delta ABC \sim \Delta DEF$

i.e $\angle A \cong \angle D$, $\angle B \cong \angle E$ and $\angle C \cong \angle F$

To Prove

$$\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}}$$

Construction

(I) Suppose that $m\overline{AB} > m\overline{DE}$

(II) $m\overline{AB} \leq m\overline{DE}$

On \overline{AB} take a point L such that $m\overline{AL} = m\overline{DE}$

On \overline{AC} take a point M such that $m\overline{AM} = m\overline{DF}$

Join L and M by the line segment LM

Proof

Statements	Reasons
In $\Delta ALM \leftrightarrow \Delta DEF$	
$\angle A \cong \angle D$	Given
$\overline{AL} \cong \overline{DE}$	Construction
$\overline{AM} \cong \overline{DF}$	Construction
Thus $\Delta ALM \cong \Delta DEF$	S.A.S Postulate
And $\angle L \cong \angle E$, $\angle M \cong \angle F$	(Corresponding angles of congruent triangles)
Now $\angle E \cong \angle B$ and $\angle F \cong \angle C$	Given
$\therefore \angle L \cong \angle B$, $\angle M \cong \angle C$	Transitivity of congruence
Thus $\overline{LM} \parallel \overline{BC}$	Corresponding angles are equal
Hence $\frac{m\overline{AL}}{m\overline{AB}} = \frac{m\overline{AM}}{m\overline{AC}}$	A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.
Or $\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{DF}}{m\overline{AC}}$(i)	$m\overline{AL} = m\overline{DE}$ and $m\overline{AM} = m\overline{DF}$ (Construction)
Similarly by intercepting segments on \overline{BA} and \overline{BC} , we can prove that	
$\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{EF}}{m\overline{BC}}$(ii)	

$$\text{Thus } \frac{\overline{mDE}}{\overline{mAB}} = \frac{\overline{mDF}}{\overline{mAC}} = \frac{\overline{mEF}}{\overline{mBC}}$$

$$\text{Or } \frac{\overline{mAB}}{\overline{mDE}} = \frac{\overline{mAC}}{\overline{mDF}} = \frac{\overline{mBC}}{\overline{mEF}}$$

$$\text{If } \overline{mAB} = \overline{mDE}$$

Then in $\triangle ABC \leftrightarrow \triangle DEF$

(II) If $\overline{mAB} < \overline{mDE}$, it can similarly be proved by taking intercepts on the sides of $\triangle DEF$

$$\angle A \cong \angle D$$

$$\angle B \cong \angle E$$

$$\text{And } \overline{AB} \cong \overline{DE}$$

So $\triangle ABC \cong \triangle DEF$

$$\text{Thus } \frac{\overline{mAB}}{\overline{mDE}} = \frac{\overline{mAC}}{\overline{mDF}} = \frac{\overline{mBC}}{\overline{mEF}} = 1$$

Hence the result is true for all the cases.

By (i) and (ii)

By taking reciprocals

A.S.A \cong A.S.A

$$\overline{AC} \cong \overline{DF}, \quad \overline{BC} \cong \overline{EF}$$

Exercise 14.2

Q.1 In $\triangle ABC$ as shown in the figure \overline{CD} bisects $\angle C$ and meets \overline{AB} at D. $m\overline{BD}$ is equal to

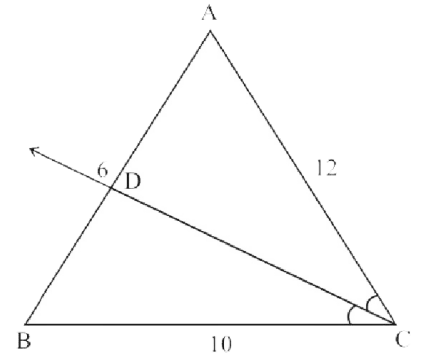
- (a) 5 (b) 16 (c) 10 (d) 18

$$\frac{m\overline{BD}}{m\overline{DA}} = \frac{m\overline{BC}}{m\overline{CA}}$$

$$\frac{\overline{BD}}{6} = \frac{10}{12}$$

$$\overline{BD} = \frac{10^5 \times 6^2}{12^4_2} \text{ or } \overline{BD} = \frac{10 \times 6}{12} = \frac{60^5}{12}$$

$$\overline{BD} = 5$$



Q.2 In $\triangle ABC$ shown in the figure \overline{CD} bisects $\angle C$. If $m\overline{AC}=3$, $\overline{CB}=6$ and $m\overline{AB}=7$ then find $m\overline{AD}$ and \overline{DB}

$$\overline{AB} = \overline{AD} + \overline{BD}$$

$$\overline{AD} = \overline{AB} - \overline{BD}$$

$$\overline{AD} = 7 - x$$

$$\frac{m\overline{AD}}{m\overline{BD}} = \frac{m\overline{AC}}{m\overline{CB}}$$

$$\frac{x}{7-x} = \frac{3}{6}$$

$$\frac{x}{7-x} = \frac{1}{2}$$

$$2x = 7-x$$

$$2x + x = 7$$

$$3x = 7$$

$$x = \frac{7}{3} \text{ or } \overline{AD} = \frac{7}{3}$$

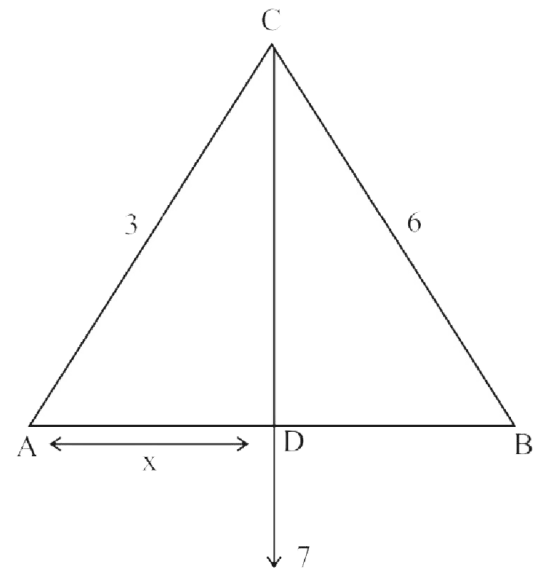
$$\overline{AB} = \overline{AD} + \overline{BD}$$

$$7 = \frac{7}{3} + \overline{BD}$$

$$7 - \frac{7}{3} = \overline{BD}$$

$$\frac{21-7}{3} = \overline{BD}$$

$$\overline{BD} = \frac{14}{3}$$



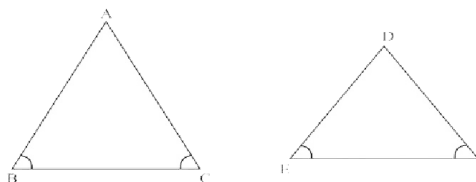
Q.3 Show that in any corresponding of two triangles if two angles of one triangle are congruent to the corresponding angles of the other, then the triangle are similar

Given

$\triangle ABC$ and $\triangle DEF$

$\angle B \cong \angle E$

$\angle C \cong \angle F$



To Prove

$\triangle ABC \cong \triangle DEF$

Proof

Statements	Reasons
$\angle A + \angle B + \angle C = 180^\circ$	Sum of three angles of a triangle = 180°
$\angle D + \angle E + \angle F = 180$	
$\angle A \cong \angle D$	
$\angle B = \angle E$	
$\angle C = \angle F$	
Hence $\triangle ABC \cong \triangle DEF$	

Q.4 If line segment \overline{AB} and \overline{CD} are intersecting at point X and $\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}}$ then

show that $\triangle AXC$ and $\triangle BXD$ are similar

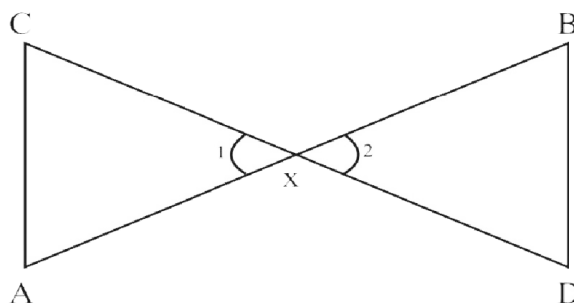
Given

Line segment \overline{AB} and \overline{CD} intersect at X

$$\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}}$$

To Prove

$\triangle CXA$ and $\triangle DXB$ are similar



Proof

Statements	Reasons
$\frac{\overline{AX}}{\overline{XB}} = \frac{\overline{CX}}{\overline{XD}}$	Given
$\angle 1 \cong \angle 2$	
$\overline{AC} \parallel \overline{BD}$	Vertical angles
$\angle A = m\angle B$	Alternate angles
$m\angle C = m\angle D$	
Hence proved the triangle are similar	

Review Exercise 14

Q.1 Which of the following are true which are false?

- (i) Congruent triangles are of same size and shape. (True)
- (ii) Similar triangles are of same shape but different sizes. (True)
- (iii) Symbol used for congruent is ‘ \sim ’ (False)
- (iv) Symbol used for similarity is ‘ \cong ’ (False)
- (v) Congruent triangle are similar (True)
- (vi) Similar triangles are congruent (False)
- (vii) A line segment has only one midpoint (True)
- (viii) One and only one line can be drawn through two points (True)
- (ix) Proportion is non equality of two ratio (False)
- (x) Ratio has no unit (True)

Q.2 Define the following

(i) Ratio

The ratio between two a like quantities is defined as $a : b = \frac{a}{b}$ where a and are the elements of the ratio.

(ii) Proportion

Proportion is defined as the equality of two ratio i,e $a : b = c : d$

(iii) Congruent Triangles

Two triangles are said to be congruent (symbols \cong) if there emits a corresponding between them such that all the corresponding sides and angles are congruent.

(iv) Similar Triangles

If two triangles are similar then the measures of their corresponding sides are proportional.

Q.3 In $\triangle LMN$ shown in the figure $\overline{MN} \parallel \overline{PQ}$

- (i)** If $m\overline{LM} = 5\text{cm}$, $m\overline{LP} = 2.5\text{cm}$
 $m\overline{LQ} = 2.3 \text{ cm}$ then find LN

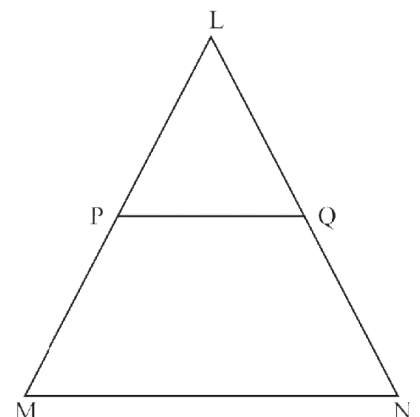
$$\frac{m\overline{LP}}{m\overline{LM}} = \frac{m\overline{LQ}}{m\overline{LN}}$$

$$\frac{2.5}{5} = \frac{2.3}{\overline{LN}}$$

$$(2.5) \overline{LN} = 5 \times 2.3$$

$$\overline{LN} = \frac{11.5}{2.5}$$

$$\overline{LN} = 4.6\text{cm}$$



- (ii) If $mLM = 6\text{cm}$, $mLQ = 2.5\text{cm}$
 $mQN = 5\text{cm}$ then find
 mLP

$$\frac{mLP}{mLM} = \frac{mLQ}{mLN}$$

$$\frac{LP}{6} = \frac{2.5}{LN}$$

$$\overline{LN} = \overline{LQ} + \overline{QN}$$

$$\overline{LN} = 2.5 + 5$$

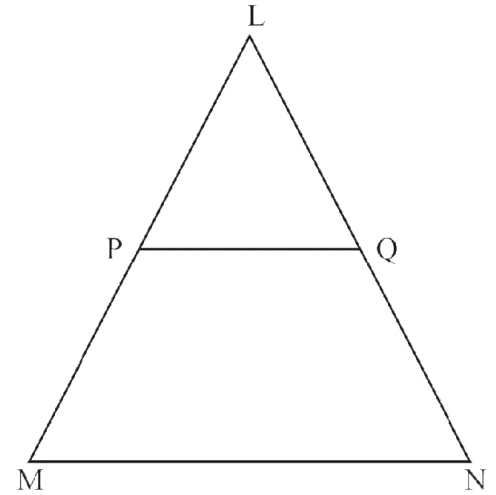
$$\overline{LN} = 7.5\text{cm}$$

$$\frac{\overline{LP}}{6} = \frac{2.5}{7.5}$$

$$\overline{LP} = \frac{2.5 \times 6}{7.5}$$

$$\overline{LP} = \frac{15}{7.5}$$

$$\overline{LP} = 2\text{cm}$$



- Q.4** In the show figure let $mPA = 8x - 7$ $mPB = 4x - 3$ $mAQ = 5x - 3$
 $mBR = 3x - 1$ find the value of x if $\overline{AB} \parallel \overline{QR}$

$$\frac{mPA}{mAQ} = \frac{mBP}{mBR}$$

$$\frac{8x - 7}{5x - 3} = \frac{4x - 3}{3x - 1}$$

By cross multiplying

$$(8x - 7)(3x - 1) = (4x - 3)(5x - 3)$$

$$24x^2 - 8x - 21x + 7 = 20x^2 - 12x - 15x + 9$$

$$24x^2 - 29x + 7 = 20x^2 - 27x + 9$$

$$24x^2 - 20x^2 - 29x + 27x + 7 - 9 = 0$$

$$4x^2 - 2x - 2 = 0$$

$$4x^2 - 4x + 2x - 2 = 0$$

$$4x(x - 1) + 2(x - 1) = 0$$

$$(x - 1)(4x + 2) = 0$$

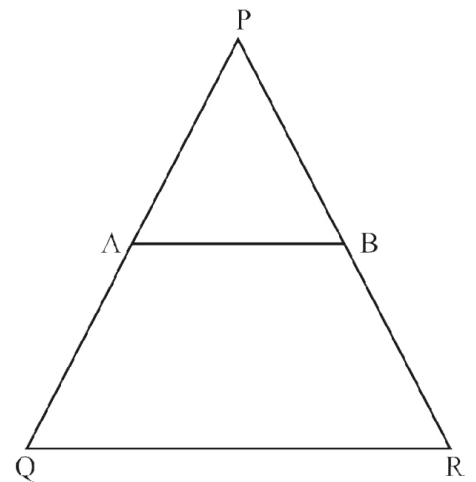
$$x - 1 = 0$$

$$x = 1$$

$$4x + 2 = 0$$

$$4x = -2$$

$$x = \frac{-2}{4}$$



$$x = \frac{-1}{2}$$

Length is always taken as positive not negative so value of $x = 1$

Q.5 In $\triangle LMN$ Shown in figure \overline{LA} bisects $\angle L$. If $m\overline{LN} = 4m$ $m\overline{LM} = 6cm$ $m\overline{MN} = 8$ then find

$m\overline{MA}$ and $m\overline{AN}$

$$\frac{m\overline{MA}}{m\overline{AN}} = \frac{m\overline{LM}}{m\overline{LN}}$$

$$\overline{MA} = x$$

$$\overline{AN} = 8-x$$

$$\frac{x}{8-x} = \frac{6}{4}$$

$$4x = 6(8-x)$$

$$4x = 48 - 6x$$

$$4x + 6x = 48$$

$$10x = 48$$

$$x = \frac{48}{10}$$

$$x = 4.8cm$$

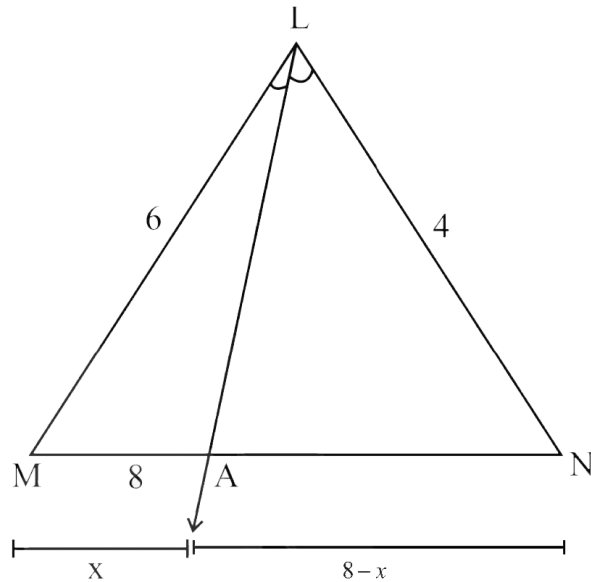
$$m\overline{MA} = 4.8cm$$

$$\overline{MN} = \overline{MA} + \overline{AN}$$

$$8 = 4.8 + \overline{AN}$$

$$8 - 4.8 = \overline{AN}$$

$$\overline{AN} = 3.2cm$$



Q.6 In Isosceles $\triangle PQR$ Shown in the figure, find the value of x and y

As we know that it is isosceles triangle

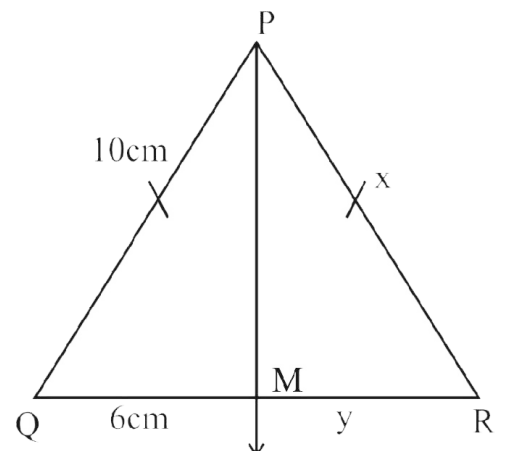
So

$$\overline{PQ} = \overline{RP}$$

$$10 = x$$

Or

$$x = 10cm$$



$$\overline{PM} \perp \overline{QR}$$

So it bisects the side and bisects the angle also

$$\text{SO } \overline{QM} = \overline{MR}$$

$$6 = y$$

Or

$$y = 6\text{cm}$$

Unit 14: Ratio and Proportion

Overview

Theorem 14.1.1

A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.

Given:

In $\triangle ABC$, the line ℓ is intersecting the sides \overline{AC} and \overline{AB} at points E and D respectively such that $\overline{ED} \parallel \overline{CB}$

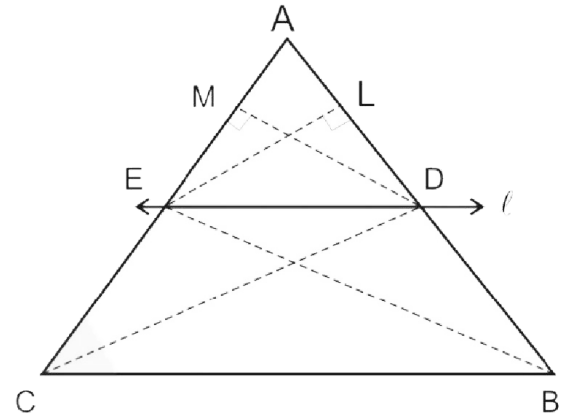
To Prove

$$m\overline{AD} : \overline{DB} = m\overline{AE} : \overline{EC}$$

Construction:

Join B to E and C to D. From D draw $\overline{DM} \perp \overline{AC}$ and from E draw $\overline{EL} \perp \overline{AB}$

Proof



Statements	Reasons
In triangles BED and AED, EL is the common perpendicular	
\therefore Area of $\triangle BED = \frac{1}{2} \times m\overline{BD} \times m\overline{EL} \dots \dots (i)$	Area of a $\Delta = \frac{1}{2}$ (base)(height)
and Area of $\triangle AED = \frac{1}{2} \times m\overline{AD} \times m\overline{EL} \dots \dots (ii)$	
Thus Area of $\frac{\triangle BED}{\triangle AED} = \frac{m\overline{DB}}{m\overline{AD}} \dots \dots (iii)$	Dividing (i) by (ii)
Similarly	
$\frac{\text{Area of } \triangle CDE}{\text{Area of } \triangle ADE} = \frac{m\overline{EC}}{m\overline{AE}} \dots \dots (iv)$	
But $\triangle BED \cong \triangle CDE$	(Areas of triangles with common base and same altitudes are equal. Given that $\overline{ED} \parallel \overline{CB}$, so altitudes are equal).
\therefore From (iii) and (iv) We have	
$\frac{m\overline{DB}}{m\overline{AD}} = \frac{m\overline{EC}}{m\overline{AE}}$ or	
$\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$	Taking reciprocal of both sides.
Hence $m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$	

Theorem: 14.1.2 Converse of Theorem 14.1.1

If a line segment intersects the two sides of a triangle in the same ratio, then it is parallel to the third side.

Given

In $\triangle ABC$, \overline{ED} intersect \overline{AB} and \overline{AC} such that $m\overline{AD} : \overline{DB} = m\overline{AE} : \overline{EC}$

To Prove

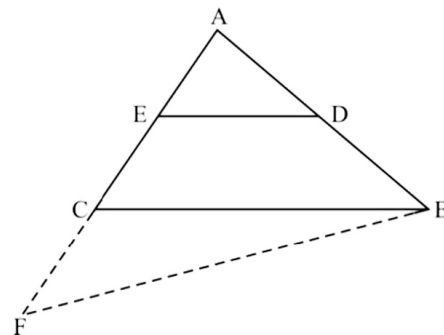
$\overline{ED} \parallel \overline{CB}$

Construction

If $\overline{ED} \not\parallel \overline{CB}$ then draw $\overline{BF} \parallel \overline{DE}$ to meet \overline{AC}

Produced at F

Proof



Statements	Reasons
In $\triangle ABF$ $\overline{DE} \parallel \overline{BF}$	Construction
$\therefore \frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EF}}$(i)	(A line parallel to one side of a triangle divides the other two sides proportionally Theorem 14.1.1)
But $\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$(ii)	Given
$\therefore \frac{m\overline{AE}}{m\overline{EF}} = \frac{m\overline{AE}}{m\overline{EC}}$ or $m\overline{EF} = m\overline{EC}$,	From (i) and (ii)
This is possible only if point F is coincident with C.	(Property of real numbers)
\therefore Our supposition is wrong	
Hence $\overline{ED} \parallel \overline{CB}$	