

Exercise 3.1

Q.1 Express each of the following numbers in scientific notations.

- (i) 5700
 $= 5.7 \times 10^3$ **Ans**
- (ii) 49,800,000
 $= 4.98 \times 10^7$ **Ans**
- (iii) 96000000
 $= 9.6 \times 10^7$ **Ans**
- (iv) 416.9
 $= 4.169 \times 10^2$ **Ans**
- (v) 83000
 $= 8.3 \times 10^4$ **Ans**
- (vi) 0.00643
 $= 6.43 \times 10^{-3}$ **Ans**
- (vii) 0.0074
 $= 7.4 \times 10^{-3}$ **Ans**
- (viii) 60,000,000
 $= 6 \times 10^7$ **Ans**
- (ix) 0.00000000395
 $= 3.95 \times 10^{-9}$ **Ans**
- (x) $\frac{275000}{0.0025}$
 $= \frac{2.75 \times 10^5}{2.5 \times 10^{-3}}$ **Ans**

Q.2 Express the following number in ordinary notation.

- (i) 6×10^{-4}
 $= 0.0006$ **Ans**
- (ii) 5.06×10^{10}
 $= 50600000000$ **Ans**
- (iii) 9.018×10^{-6}
 $= 0.000009018$ **Ans**
- (iv) 7.865×10^8
 $= 786500000$ **Ans**

Exercise 3.2

Q.1 Find the common logarithms of each of the following numbers.

(i) 232.92

Solution: 232.92

Suppose $x = 232.92$

Taking log

$\log x = \log 232.92$

$Ch = 2$

Mantissa = 0.3672

$\log x = 2.3672$ **Ans**

(ii) 29.326

Solution: 29.326

Suppose $x = 29.326$

Taking log

$\log x = \log 29.326$

$Ch = 1$

Mantissa = 0.4672

$\log x = 1.4672$ **Ans**

(iii) 0.00032

Solution: 0.00032

Suppose $x = 0.00032$

Taking log

$\log x = \log 0.00032$

$Ch = \bar{4}$

Mantissa = 0.5051

$\log x = \bar{4}.5051$ **Ans**

(iv) 0.3206

Solution: 0.3206

Suppose $x = 0.3206$

Taking log:

$\log x = \log 0.3206$

$Ch = \bar{1}$

Mantissa = 0.5059

$\log x = \bar{1}.5059$ **Ans**

Q.2 If $\log 31.09 = 1.4926$, find the value of the following.

If

$\log 31.09 = 1.4926$

Then

(i) $\log 3.109 = 0.4926$

(ii) $\log 310.9 = 2.4926$

(iii) $\log 0.003109 = \bar{3}.4926$

(iv) $0.3109 = \bar{1}.4926$

Solution:

(i) $\log 3.109$

Characteristics = 0

Mantissa = 0.4926

$\log 3.109 = 0.4926$ **Ans**

(ii) $\log 310.9$

Characteristics = 2

Mantissa = 0.4926

$\log 310.9 = 2.4926$ **Ans**

(iii) $\log 0.003109$

Characteristics = $\bar{3}$

Mantissa = 0.4926

$\log 0.003109 = \bar{3}.4926$ **Ans**

(iv) $\log 0.3109$

Characteristics = $\bar{1}$

Mantissa = 0.4926

$\log 0.3109 = \bar{1}.4926$ **Ans**

Q.3 Find the numbers whose common logarithms are

(i) 3.5621

Solution:

$\log x = 3.5621$

$Ch = 3$ (If ch is positive, then plus for reference point)

Mantissa = 0.5621

$x = \text{antilog } 3.5621$

$x = 3649.0$ **Ans**

(ii) $\bar{1}.7427$

Solution:

$\log x = \bar{1}.7427$

$Ch = \bar{1}$

Mantissa = 0.7427

$$x = \text{anti log } \bar{1}.7427$$

$$x = 0.5530 \text{ Ans}$$

Q.4 What replacement for the unknown in each of the following will make the true statements?

(i) $\log_3 81 = L$

Solution: $\log_3 81 = L$

Writing in exponential form.

$$3^L = 81$$

$$3^L = 3^4$$

\therefore Bases are equal so

$$L = 4 \text{ Ans}$$

(ii) $\log_a 6 = 0.5$

Solution: $\log_a 6 = 0.5$

$$a^{0.5} = 6$$

$$a^{\frac{1}{2}} = 6$$

$$\sqrt{a} = 6 \text{ Taking square on both}$$

sides

$$\sqrt{(a)^2} = (6)^2$$

$$a = 36 \text{ Ans}$$

(iii) $\log_5 n = 2$

Write in exponential form

$$5^2 = n$$

$$25 = n$$

$$\text{Or } n = 25 \text{ Ans}$$

(iv) $10^p = 40$

Solution: $10^p = 40$

Changing into logarithmic form

$$p = \log_{10} 40$$

$$= \log 40$$

$$= 1.6021 \text{ Ans}$$

Q.5 Evaluate.

(i) $\log_2 \frac{1}{128}$

Solution: $\log_2 \frac{1}{128}$

Suppose $\log_2 \frac{1}{128} = x$

Writing in exponential form.

$$2^x = \frac{1}{128}$$

$$2^x = \frac{1}{2^7}$$

$$2^x = 2^{-7}$$

\therefore Bases are equal so

$$x = -7 \text{ Ans}$$

(ii) **log 512 to the base $2\sqrt{2}$**

Solution: $\log_{2\sqrt{2}} 512 = x$

Writing in exponential form

$$(2\sqrt{2})^x = 512$$

$$\left(2^1 \cdot 2^{\frac{1}{2}}\right)^x = 2^9$$

$$\left(2^{\frac{3}{2}}\right)^x = 2^9$$

$$2^{\frac{3}{2}x} = 2^9$$

\therefore Bases are equal so

$$\frac{3}{2}x = 9$$

$$x = \frac{9 \times 2}{3}$$

$$x = \frac{18}{3}$$

$$x = 6 \text{ Ans}$$

Q.6 Find the value of x from the following statements.

(i) $\log_2 x = 5$

Solution: $\log_2 x = 5$

Write in exponential form.

$$2^5 = x$$

$$32 = x \text{ Ans}$$

(ii) $\log_{81} 9 = x$

Solution: $\log_{81} 9 = x$

Writing in the exponential form.

$$81^x = 9$$

$$(9^2)^x = 9$$

$$9^{2x} = 9$$

$$2x = 1$$

$$x = \frac{1}{2} \text{ Ans}$$

(iii) $\log_{64} 8 = \frac{x}{2}$

Solution: $\log_{64} 8 = \frac{x}{2}$

Writing in exponential form.

$$64^{\frac{x}{2}} = 8$$

$$(8^2)^{\frac{x}{2}} = 8$$

$$8^x = 8$$

$$x = 1 \text{ Ans}$$

(iv) $\log_x 64 = 2$

Solution: $\log_x 64 = 2$

Writing in exponential form

$$x^2 = 64$$

$$x^2 = 8^2$$

$$x = 8 \text{ Ans}$$

(v) $\log_3 x = 4$

Solution: $\log_3 x = 4$

$$3^4 = x$$

$$81 = x$$

$$\text{Or } x = 81 \text{ Ans}$$

Exercise 3.3

Q.1 Write the following into sum or difference $\log(A \times B)$

(i) $\log(A \times B)$

Solution: $\log(A \times B)$

$$\log A \times B = \log A + \log B \quad \text{Ans}$$

(ii) $\log \frac{15.2}{30.5}$

Solution: $\log \frac{15.2}{30.5}$

$$\log \frac{15.2}{30.5} = \log 15.2 - \log 30.5 \quad \text{Ans}$$

(iii) $\log \frac{21 \times 5}{8}$

Solution: $\log \frac{21 \times 5}{8}$

$$\begin{aligned} \log \frac{21 \times 5}{8} &= \log(21 \times 5) - \log 8 \\ &= \log 21 + \log 5 - \log 8 \quad \text{Ans} \end{aligned}$$

(iv) $\log \sqrt[3]{\frac{7}{15}}$

Solution: $\log \sqrt[3]{\frac{7}{15}}$

$$\begin{aligned} \log \sqrt[3]{\frac{7}{15}} &= \log \left(\frac{7}{15} \right)^{\frac{1}{3}} \\ &= \frac{1}{3} \log \left(\frac{7}{15} \right) \\ &= \frac{1}{3} (\log 7 - \log 15) \\ &= \frac{1}{3} \log 7 - \frac{1}{3} \log 15 \quad \text{Ans} \end{aligned}$$

(v) $\log \frac{(22)^{\frac{1}{3}}}{5^3}$

Solution: $\log \frac{(22)^{\frac{1}{3}}}{5^3}$

$$\begin{aligned} \log \frac{(22)^{\frac{1}{3}}}{5^3} &= \log 22^{\frac{1}{3}} - \log 5^3 \\ &= \frac{1}{3} \log 22 - 3 \log 5 \quad \text{Ans} \end{aligned}$$

(vi) $\log \frac{25 \times 97}{29}$

Solution: $\log \frac{25 \times 97}{29}$

$$\begin{aligned} \log \frac{25 \times 47}{29} &= \log(25 \times 47) - \log 29 \\ &= \log 25 + \log 47 - \log 29 \quad \text{Ans} \end{aligned}$$

Q.2 Express

$\log x - 2 \log x + 3 \log(x+1) - \log(x^2 - 1)$ as a single logarithm.

Solution:

$$\begin{aligned} \log x - 2 \log x + 3 \log(x+1) - \log(x^2 - 1) \\ = \log x - \log x^2 + \log(x+1)^3 - \log(x^2 - 1) \end{aligned}$$

$$= \log \left(\frac{x}{x^2} \right) + \log \frac{(x+1)^3}{x^2 - 1}$$

$$= \log \left(\frac{x}{x^2} \times \frac{(x+1)^3}{x^2 - 1} \right)$$

$$= \log \left(\frac{x(x+1)^3}{x^2(x^2 - 1)} \right)$$

$$= \log \frac{\cancel{x}(x+1)^2 \cancel{(x+1)}}{x \times \cancel{x}(x-1) \cancel{(x+1)}}$$

$$= \log \frac{(x+1)^2}{x(x-1)} \quad \text{Ans}$$

Q.3 Write the following in the form of a single logarithm.

(i) $\log 21 + \log 5$

Solution: $\log 21 + \log 5$
 $= \log(21 \times 5)$ **Ans**

(ii) $\log 25 - 2 \log 3$

Solution: $\log 25 - 2 \log 3$
 $= \log 25 - 2 \log 3$
 $= \log 25 - \log 3^2$
 $= \log \frac{25}{3^2}$ **Ans**

(iii) $2 \log x - 3 \log y$

Solution: $2 \log x - 3 \log y$
 $= 2 \log x - 3 \log y$
 $= \log x^2 - \log y^3$
 $= \log \frac{x^2}{y^3}$ **Ans**

(iv) $\log 5 + \log 6 - \log 2$

Solution: $\log 5 + \log 6 - \log 2$
 $= \log 5 + \log 6 - \log 2$
 $= \log(5 \times 6) - \log 2$
 $= \log \frac{5 \times 6}{2}$ **Ans**

Q.4 Calculate the following.

(i) $\log_3 2 \times \log_2 81$

Solution: $\log_3 2 \times \log_2 81$
 $= \frac{\cancel{\log 2}}{\log 3} \times \frac{\log 81}{\cancel{\log 2}}$
 $= \frac{\log 81}{\log 3}$
 $= \frac{\log 3^4}{\log 3}$
 $= \frac{4 \cancel{\log 3}}{\cancel{\log 3}}$
 $= 4$ **Ans**

(ii) $\log_3 \times \log_3 25$

Solution: $\log_3 \times \log_3 25$
 $= \frac{\cancel{\log 3}}{\log 5} \times \frac{\log 25}{\cancel{\log 3}}$
 $= \frac{\log 25}{\log 5}$
 $= \frac{\log 5^2}{\log 5}$
 $= \frac{2 \cancel{\log 5}}{\cancel{\log 5}}$
 $= 2$ **Ans**

Q.5 If $\log 2 = 0.3010$, $\log 3 = 0.4771$ and $\log 5 = 0.6990$, then find the values of the following.

(i) $\log 32$

$= \log 2^5$

\therefore using 3rd law of logarithm

$= 5 \log 2$

By putting the value of $\log 2$

$= 5(0.3010)$

$= 1.5050$ **Ans**

(ii) $\log 24$

Solution: $\log 24$

$= \log(2^3 \times 3)$

$= \log 2^3 + \log 3$

$= 3 \log 2 + \log 3$

By putting the value of $\log 2$ and $\log 3$

$= 3(0.3010) + 0.4771$

$= 0.9030 + 0.4771$

$= 1.3801$ **Ans**

(iii) $\log \sqrt{3 \frac{1}{3}}$

Solution: $\log \sqrt{3 \frac{1}{3}}$
 $= \log \left(\frac{10}{3} \right)^{\frac{1}{2}}$

$$= \frac{1}{2} \log \left[\frac{2 \times 5}{3} \right]$$

$$= \frac{1}{2} (\log 2 + \log 5 - \log 3)$$

By putting the values of $\log 2, \log 3$ and $\log 5$

$$= \frac{1}{2} (0.3010 + 0.69900 - 0.4771)$$

$$= \frac{1}{2} (1 - 0.4771)$$

$$= \frac{1}{2} (0.5229)$$

$$= 0.26145 \text{ Ans}$$

(iv) $\log \frac{8}{3}$

Solution: $\log \frac{8}{3}$

$$= \log \frac{2^3}{3}$$

$$= \log 2^3 - \log 3$$

$$= 3 \log 2 - \log 3$$

By putting the values of $\log 2$ and $\log 3$

$$= 3(0.3010) - 0.4771$$

$$= 0.9030 - 0.4771$$

$$= 0.4259 \text{ Ans}$$

(v) $\log 30$

Solution: $\log 30$

$$= \log (5 \times 2 \times 3)$$

\therefore using first law of logarithm

$$= \log 5 + \log 2 + \log 3$$

By putting the values of $\log 2, \log 3$ $\log 5$

$$= (0.6990) + (0.3010) + (0.4771)$$

$$= 1.4771 \text{ Ans}$$

Exercise 3.4

Q.1 Use log tables to find the value of

(i) 0.8176×13.64

Solution: 0.8176×13.64

Suppose

$$x = 0.8176 \times 13.64$$

Taking log on both sides

$$\log x = \log(0.8176 \times 13.64)$$

According to first law of logarithm

$$\log x = \log 0.8176 + \log 13.64$$

$$= \bar{1}.9125 + \underline{1}.1348$$

$$\log x = -1 + 0.9125 + 1.1348$$

$$\log x = 1.0473$$

To find antilog

$$x = \text{antilog } 1.0473$$

$$\text{Ch} = 1$$

$$x = 1.115$$

Reference point

$$x = 11.15 \text{ Ans}$$

(ii) $(789.5)^{\frac{1}{8}}$

Solution: $(789.5)^{\frac{1}{8}}$

Let $x = (789.5)^{\frac{1}{8}}$

Taking log on both sides

$$\log x = \log (789.5)^{\frac{1}{8}}$$

According to third law

$$\log x = \frac{1}{8} \log (789.5)$$

$$\log x = \frac{1}{8} (2.8974)$$

$$= \frac{2.8974}{8}$$

$$\log x = 0.3622$$

To find antilog

$$x = \text{antilog } 0.3622$$

$$\text{Characteristics} = 0$$

$$x = 2.302$$

Reference point

$$x = 2.302 \text{ Ans}$$

(iii) $\frac{0.678 \times 9.01}{0.0234}$

Solution: $\frac{0.678 \times 9.01}{0.0234}$

Suppose

$$x = \frac{0.678 \times 9.01}{0.0234}$$

Taking log on both sides

$$\log x = \log \frac{0.678 \times 9.01}{0.0234}$$

According to 1st and 2nd law of log

$$\log x = \log 0.678 + \log 9.01 - \log 0.0234$$

$$\log x = \bar{1}.8312 + 0.9547 - \bar{2}.3692$$

$$= -1 + 0.8312 + 0.9547 - (-2 + 0.3692)$$

$$= 2.4167$$

To find antilog

$$x = \text{antilog } 2.4167$$

$$\text{Characteristics} = 2$$

$$x = 2.610$$

$$x = 261.0 \text{ Ans}$$

(iv) $\sqrt[5]{2.709} \times \sqrt[7]{1.239}$

Solution: $\sqrt[5]{2.709} \times \sqrt[7]{1.239}$

$$(2.709)^{\frac{1}{5}} \times (1.239)^{\frac{1}{7}}$$

Suppose:

$$x = (2.709)^{\frac{1}{5}} \times (1.239)^{\frac{1}{7}}$$

Taking log on both side

$$\log x = \log \left[(2.709)^{\frac{1}{5}} \times (1.239)^{\frac{1}{7}} \right]$$

According to law of logarithm

$$\log x = \log (2.709)^{\frac{1}{5}} + \log (1.239)^{\frac{1}{7}}$$

According to third law of logarithm

$$\log x = \frac{1}{5} \log (2.709) + \frac{1}{7} \log (1.239)$$

$$\log x = \frac{1}{5} \log (2.709) + \frac{1}{7} \log (1.239)$$

$$= \frac{1}{5} 0.4328 + \frac{1}{7} 0.0931$$

$$= \frac{0.4328}{5} + \frac{0.0931}{7}$$

$$0.0866 + 0.0133$$

$$= 0.0999$$

To find antilog

$$x = \text{antilog } 0.999$$

Characteristics = 0

$$x = 1.259$$

Reference point

$$x = 1.259 \text{ Ans}$$

$$(v) \frac{1.23 \times 0.6975}{0.0075 \times 1278}$$

$$\text{Solution: } \frac{1.23 \times 0.6975}{0.0075 \times 1278}$$

Suppose

$$x = \frac{1.23 \times 0.6975}{0.0075 \times 1278}$$

$$\log x = \log \frac{1.23 \times 0.6975}{0.0075 \times 1278}$$

$$= \log(1.23 \times 0.6975) - \log(0.0075 \times 1278)$$

$$= \log 1.23 + \log 0.6975 - (\log 0.0075 + \log 1278)$$

$$= \log 1.23 + \log 0.6975 - \log 0.0075 - \log 1278$$

$$= 0.0899 + \bar{1}.8435 - \bar{3}.8751 - 3.1065$$

$$= 0.8999 + (-1 + 0.8435) - (-3 + 0.8751) + 3.1065$$

$$= -1.0482$$

$$\log x = -2 + 2 - 1.0482$$

$$\log x = 02 + 0.9515$$

$$\log x = \bar{2}.9518$$

To find antilog

$$x = \text{antilog } \bar{2}.9518$$

$$\text{Ch} = \bar{2}$$

$$x = 8950$$

$$= 0.08950 \text{ Ans}$$

$$(vi) \sqrt[3]{\frac{0.7214 \times 20.37}{60.8}}$$

$$\text{Solution: } \sqrt[3]{\frac{0.7214 \times 20.37}{60.8}}$$

$$\text{Let } x = \left[\frac{0.7214 \times 20.37}{60.8} \right]^{\frac{1}{3}}$$

Taking log on both sides

$$\log x = \log \left(\frac{0.7214 \times 20.37}{60.8} \right)^{\frac{1}{3}}$$

3rd of logarithm

$$\log x = \frac{1}{3} \log \left[\frac{0.7214 \times 20.37}{60.8} \right]$$

According to first and 2nd law

$$\log x = \frac{1}{3} [\log 0.7214 + \log 37 - \log 60.8]$$

$$\log x = \frac{1}{3} [\bar{1}.8582 + 1.3089 - 1.7839]$$

$$\frac{1}{3} [-1 + 0.8582 + 1.3089 - 1.7839]$$

$$= \frac{1}{3} (-0.6168)$$

$$= -0.2056$$

$\log x$ is in negative, so

$$\log x = -1 + 1 - 0.2056$$

$$= -1 + 79144$$

$$= \bar{1}.7944$$

To find antilog

$$x = \text{antilog } \bar{1}.7944$$

$$\text{Ch} = \bar{1}$$

$$x = 6229$$

Reference point

$$0.6229 \text{ Ans}$$

$$(vii) \frac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$$

$$\text{Solution: } \frac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$$

$$\text{Suppose: } x = \frac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$$

$$x = \frac{83 \times (92)^{\frac{1}{3}}}{127 \times (246)^{\frac{1}{5}}}$$

Taking on both side

$$\log x = \log \frac{83 \times (92)^{\frac{1}{3}}}{127 \times (246)^{\frac{1}{5}}}$$

According to 1st and 2nd law of log

$$\log x = \log 83 + \log (92)^{\frac{1}{3}} - \log 127 - \log (246)^{\frac{1}{5}}$$

According to third law of log

$$\log x = \log 83 + \frac{1}{3} \log 92 - \log 127 - \frac{1}{5} \log 246$$

$$\log x = (1.9191) + \frac{1}{3}(1.9638) - (2.1038)$$

$$- \frac{1}{5}(2.3909)$$

$$= 1.9191 + 0.65460 - 2.1038 - 0.47818$$

$$= 1.9191 + 0.6546 - 2.1038 - 0.47818$$

$$= -0.0083$$

$\log x$ is in negative, so

$$\log x = -1 + 1 - 0.0083$$

$$= -1 + 0.9917$$

$$= \bar{1}.9917$$

To find antilog

$$x = \text{antilog } \bar{1}.9917$$

$$\text{Ch} = \bar{1}$$

$$x = 9.811$$

Reference point

$$x = 0.9811 \text{ Ans}$$

$$\text{(viii)} \quad \frac{(438)^3 \sqrt{0.056}}{(388)^4}$$

$$\text{Solution: } \frac{(438)^3 \sqrt{0.056}}{(388)^4}$$

$$\text{Suppose: } x = \frac{(438)^3 \sqrt{0.056}}{(388)^4}$$

$$x = \frac{(438)^3 (0.056)^{\frac{1}{2}}}{(388)^4}$$

$$x = \frac{(438)^3 (0.056)^{\frac{1}{2}}}{(388)^4}$$

Taking log on both side

$$\log x = \log \left(\frac{(438)^3 (0.056)^{\frac{1}{2}}}{(388)^4} \right)$$

According to 1st and 2nd law

$$\log x = \log (438)^3 + \log (0.056)^{\frac{1}{2}} - \log (388)^4$$

According to third law

$$\log x = 3 \log (438) + \frac{1}{2} \log (0.056) - 4 \log (388)$$

$$\log x = 3(2.6415) + \frac{1}{2}(\bar{2}.7482) - 4(2.5888)$$

$$= 7.9245 + \frac{1}{2}(-2 + 0.7482) - 10.3552$$

$$= 7.9245 + \frac{1}{2}(-1.2518) - 10.3552$$

$$= 7.9245 - 0.6259 - 10.3552$$

$$= -3.0566$$

\log is in negative, so

$$\log x = -4 + 4 - 3.0566$$

$$= -4 + 0.9434$$

To find antilog

$$x = \text{antilog } \bar{4}.9434$$

$$\text{Ch} = \bar{4}$$

$$x = 8778$$

Reference point

$$= 0.0008778 \text{ Ans}$$

Q.2 A gas is expanding according to the law $pv^n = C$.

Find C when $p = 80$, $v = 3.1$ and

$$n = \frac{5}{4}.$$

Solution: Given that $pv^n = C$

Taking log on both sides

$$\text{Log } (pv^n) = \log C$$

$$\text{Log } P + \log v^n = \log C$$

$$\text{Log } C = \log P + \log v^n$$

$$\text{Log } C = \log P + n \log v$$

$$\text{Putting } P=80, v=3.1 \text{ and } n = \frac{5}{4}$$

$$\begin{aligned} \text{Log } C &= \log 80 + \frac{5}{4} \log 3.1 \\ &= 1.9031 + \frac{5}{4} (0.4914) \\ &= 1.9031 + 0.6143 \\ \text{Log } C &= 2.5174 \\ \text{Taking antilog both sides} \\ C &= \text{Antilog } (2.5174) \\ C &= 329.2 \text{ Ans:} \end{aligned}$$

Q.3 The formula $p = 90(5)^{-q/10}$ applies to the demand of a product, where q is the number of units and p is the price of one unit. How many units will be demanded if the price is Rs 18.00?

Solution: Given that $p = 90(5)^{-q/10}$
 Taking log on both sides

$$\text{Log } p = \log \left(90(5)^{-q/10} \right)$$

$$\text{Log } p = \log 90 + \log 5^{-q/10}$$

$$\text{Log } p = \log 90 - \frac{q}{10} \log 5$$

$$\text{Log } 18 = \log 90 - \frac{q}{10} \log 5$$
 (P = 18)

$$1.2553 = 1.9542 - \frac{q}{10} \times 0.6990$$

$$1.2553 - 1.9542 = -\frac{q}{10} \times 0.6990$$

$$-0.6989 \times 10 = -q \times 0.6990$$

$$-6.989 = -q \times 0.6990$$

$$\frac{6.989}{0.6990} = q$$

$$q = 10 \text{ approximately}$$
 Hence 10 units will be demanded

Q.4 If $A = \pi r^2$, find A, when $\pi = \frac{22}{7}$ and $r = 15$.

Solution: Given that $A = \pi r^2$
 Taking log on both sides

$$\text{Log } A = \log \pi r^2$$

$$\text{Log } A = \log \pi + \log r^2$$

$$\begin{aligned} \text{Log } A &= \log \pi + 2 \log r \\ \text{Putting } \pi &= \frac{22}{7} \text{ and } r = 15 \\ \text{Log } A &= \log \frac{22}{7} + 2 \log 15 \\ &= \log 22 - \log 7 + 2 \log 15 \\ &= 1.3424 - 0.8451 + 2(1.1761) \\ &= 0.4973 + 2.3522 \\ \text{Log } A &= 2.8495 \\ \text{Taking antilog on both sides} \\ A &= \text{antilog } 2.8495 \\ A &= 707.1 \text{ Ans} \end{aligned}$$

Q.5 If $V = \frac{1}{3} \pi r^2 h$, find V, when $\pi = \frac{22}{7}$, $r = 2.5$ and $h = 4.2$.

Solution: Given that $V = \frac{1}{3} \pi r^2 h$
 Taking log on both sides

$$\text{Log } V = \log \frac{1}{3} \pi r^2 h$$

$$= \log \frac{1}{3} + \log \pi r^2 h$$

$$= \log 1 - \log 3 + \log \pi r^2 + \log h$$

$$= 0 - 0.4771 + \log \pi + \log r^2 + \log h$$

$$= -0.4771 + \log \frac{22}{7} + 2 \log r + \log h$$

$$\left(\pi = \frac{22}{7}, r = 2.5 \text{ and } h = 4.2 \right)$$

$$= -0.4771 + \log 22 - \log 7 + 2 \log 2.5 + \log 4.2$$

$$= -0.4771 + 1.3424 - 0.8450 + 2 \times 0.3979 + 0.6232$$

$$= -0.4771 + 1.3424 - 0.8450 + 0.7959 + 0.6232$$

$$\text{Log } V = 1.4394$$
 Taking antilog on both sides

$$V = \text{antilog } 1.4394$$

$$V = 27.50 \text{ Ans}$$

Review Exercise 3

Q.1 Multiple choice Questions. Choose of the correct answer.

- (i) If $a^x = n$, then...
- (a) $a = \log_x n$ (b) $x = \log_n a$
(c) $x = \log_a n$ (d) $a = \log_n x$
- (ii) The relation $y = \log_z x$ implies...
- (a) $x^y = z$ (b) $z^y = x$
(c) $x^z = y$ (d) $y^z = x$
- (iii) The logarithm of unity to any base is...
- (a) 1 (b) 10
(c) e (d) 0
- (iv) The logarithm of any number to itself as base is...
- (a) 1 (b) 0
(c) e (d) 10
- (v) $\log e = \dots$, where $e \approx 2.718$
- (a) 0 (b) 0.4343
(c) ∞ (d) 1
- (vi) The value of $\log\left(\frac{p}{q}\right)$ is...
- (a) $\log p - \log q$ (b) $\frac{\log p}{\log q}$
(c) $\log p + \log q$ (d) $\log q - \log p$
- (vii) $\log p - \log q$ is same as ...
- (a) $\log\left(\frac{q}{p}\right)$ (b) $\log(p - q)$
(c) $\frac{\log p}{\log q}$ (d) $\log q - \log p$
- (viii) $\log(m^n)$ can be written as...
- (a) $(\log m)^n$ (b) $m \log n$
(c) $n \log m$ (d) $\log(mn)$

(ix) $\log_b a \times \log_c b$ can be written as...

- (a) $\log_a c$
 (c) $\log_a b$

- (b) $\log_c a$
 (d) $\log_b c$

(x) $\text{Log}_y x$ will be equal to...

- (a) $\frac{\log_z x}{\log_y z}$
 (c) $\frac{\log_z x}{\log_z y}$

- (b) $\frac{\log_x z}{\log_y z}$
 (d) $\frac{\log_z y}{\log_z x}$

ANSWER KEY

i	ii	iii	iv	v	vi	vii	viii	ix	x
c	b	d	a	b	a	d	c	b	c

Q.2 Complete the following:

- (i) For common logarithm, the base is...
 (ii) The integral part of the common logarithm of a number is called the ...
 (iii) The decimal part of the common logarithm of a number is called the ...
 (iv) If $x = \log y$, then y is called the... of x .
 (v) If the characteristic of the logarithm of a number have...zero(s) immediately after the decimal point.
 (vi) If the characteristic of the logarithm of a number is 1, that number will have digits in its integral part.

ANSWER KEY

i	ii	iii	iv	v	vi
10	Characteristic	Mantissa	Antilogarithm	One	2

Q.3 Find the value of x in the following.

(i) $\log_3 x = 5$

Solution: $\log_3 x = 5$

Write in exponential form.

$3^5 = x$

$243 = x$ **Ans**

(ii) $\log_4 256 = x$

Solution: $\log_4 256 = x$

Write in exponential form

$4^x = 256$

$4^x = 4^4$

$x = 4$

$x = 4$ **Ans**

(iii) $\log_{625} 5 = \frac{1}{4}x$

Solution: $\log_{625} 5 = \frac{1}{4}x$

Write in exponential form

$(625)^{\frac{1}{4}x} = 5$

$(625)^{\frac{x}{4}} = 5$

$(5^4)^{\frac{x}{4}} = 5$

$$5^{\frac{4x}{4}} = 5$$

$$5^x = 5^1$$

$$x = 1 \text{ Ans}$$

$$(iv) \log_{64} x = -\frac{2}{3}$$

$$\text{Solution: } \log_{64} x = -\frac{2}{3}$$

Write in exponential form

$$(64)^{\frac{-2}{3}} = x$$

$$(4^3)^{\frac{-2}{3}} = x$$

$$4^{\frac{-6}{3}} = x$$

$$4^{-2} = x$$

$$\frac{1}{4^2} = x$$

$$\frac{1}{16} = x \text{ Ans}$$

Q.4 Find the value of x in the following.

$$(i) \log x = 2.4543$$

$$\text{Solution: } \log x = 2.4543$$

$$\log x = 2.4543$$

$$x = \text{antilog } 2.4543$$

$$\text{Ch} = 2$$

$$x = 284.6 \text{ Ans}$$

$$(ii) \log x = 0.1821$$

$$\text{Solution: } \log x = 0.1821$$

$$\log x = 0.1821$$

$$x = \text{antilog } 0.1821$$

$$\text{Ch} = 0$$

$$x = 1.521 \text{ Ans}$$

$$(iii) \log x = 0.0044$$

$$\text{Solution: } \log x = 0.0044$$

$$\log x = 0.0044$$

$$x = \text{antilog } 0.0044$$

$$\text{Ch} = 0$$

$$x = 1.010 \text{ Ans}$$

$$(iv) \log x = \bar{1}.6238$$

$$\text{Solution: } \log x = \bar{1}.6238$$

$$\log x = \bar{1}.6238$$

$$x = \text{antilog } \bar{1}.6238$$

$$\text{Ch} = \bar{1}$$

$$x = 0.4206 \text{ Ans}$$

Q.5 If $\log 2 = 0.3010$, $\log 3 = 0.4771$, and $\log 5 = 0.6990$ then find the values of the following.

$$(i) \log 45$$

$$\text{Solution: } \log 45$$

$$= \log(9 \times 5)$$

$$= \log(3^2 \times 5)$$

$$= \log 3^2 + \log 5$$

$$= 2 \log 3 + \log 5$$

$$= 2(0.4771) + 0.6990$$

$$= 0.9542 + 0.6990$$

$$= 1.6532 \text{ Ans}$$

$$(ii) \log \frac{16}{15}$$

$$\text{Solution: } \log \frac{16}{15}$$

$$= \log \frac{2^4}{3 \times 5}$$

$$= \log 2^4 - \log(3 \times 5)$$

$$= 4 \log 2 - (\log 3 + \log 5)$$

$$= \log 2^4 - \log 3 - \log 5$$

$$= 4 \log 2 - \log 3 - \log 5$$

$$= 4(0.3010) - 0.4771 - 0.6990$$

$$= 1.2040 - 0.4771 - 0.6990$$

$$= 0.0279 \text{ Ans}$$

(iii) $\log 0.048$

Solution: $\log 0.048$

$$\begin{aligned} &= \log \frac{48}{1000} \\ &= \log \frac{2 \times 2 \times 2 \times 2 \times 3}{2 \times 2 \times 2 \times 5 \times 5 \times 5} \\ &= \log \frac{2^4 \times 3}{2^3 \times 5^3} \\ &= \log 2^4 + \log 3 - \log 2^3 - \log 5^3 \\ &= 4 \log 2 + \log 3 - 3 \log 2 - 3 \log 5 \\ &= 4(0.3010) + 0.4771 - 3(0.3010) - 3(0.6990) \\ &= 1.2040 + 0.4771 - 0.9030 - 2.0970 \\ &= -1.3189 \\ &= -1 - 0.3189 \\ &= -1 - 1 + 1 - 0.3189 \\ &= -2 + 0.6811 \\ &= \bar{2}.6811 \text{ Ans} \end{aligned}$$

Q.6 Simplify the following.

(i) $\sqrt[3]{25.47}$

Solution: $\sqrt[3]{25.47}$

Let $x = \sqrt[3]{25.47}$

$$= (25.47)^{\frac{1}{3}}$$

Taking log on both sides

$$\log x = \log (25.47)^{\frac{1}{3}}$$

$$= \frac{1}{3} \log 25.47$$

$$= \frac{1}{3} (1.4060)$$

$$\log x = 0.4687$$

$$x = \text{anti log } 0.4687$$

$$\text{Ch} = 0$$

$$x = 2.943 \text{ Ans}$$

(ii) $\sqrt[5]{342.2}$

Solution: $\sqrt[5]{342.2}$

Let

$$x = \sqrt[5]{342.2}$$

$$x = (242.)^{\frac{1}{5}}$$

Taking log on both sides

$$\log x = (342.2)^{\frac{1}{5}}$$

$$\log x = \frac{1}{5} \log 342.2$$

$$= \frac{1}{5} (2.5343)$$

$$\log x = 0.5069$$

$$\log x = \text{antilog } 0.5069$$

$$\text{Ch} = 0$$

$$x = 3.213 \text{ Ans}$$

(iii) $\frac{(8.97)^3 \times (3.95)^2}{\sqrt[3]{15.37}}$

Solution: $\frac{(8.97)^3 \times (3.95)^2}{\sqrt[3]{15.37}}$

Let $x = \frac{(8.97)^3 \times (3.95)^2}{\sqrt[3]{15.37}}$

Taking log on both sides

$$\log x = \log \frac{(8.97)^3 \times (3.95)^2}{\sqrt[3]{15.37}}$$

$$= \log (8.97)^3 + \log (3.95)^2 - \log (15.37)^{\frac{1}{3}}$$

$$= 3 \log 8.97 + 2 \log 3.95 - \frac{1}{3} \log 15.37$$

$$= 3(0.9528) + 2(0.5966) - \frac{1}{3}(1.1867)$$

$$= 2.8584 + 1.1932 - 0.3956$$

$$\log x = 3.656$$

$$x = \text{antilog } 3.656$$

$$\text{Ch} = 3$$

$$x = 4529 \text{ Ans}$$

Unit 3: Logarithms

Overview

Scientific Notation:

A number written in the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer, is called the scientific notation.

Logarithm of a Real Number:

If $a^x = y$ then x is called the logarithm of y to the base 'a' and is written as $\log_a y = x$, where $a > 0, a \neq 1$ and $y > 0$

Characteristic of logarithm of the Number:

An integral part which is positive for a number greater than 1 and negative for a number less than 1, is called the characteristic of logarithm of the number.

Mantissa of the logarithm of the Number:

A decimal part which is always positive, is called the mantissa of the logarithm of the number.

Antilogarithm:

The number whose logarithm is given is called antilogarithm.